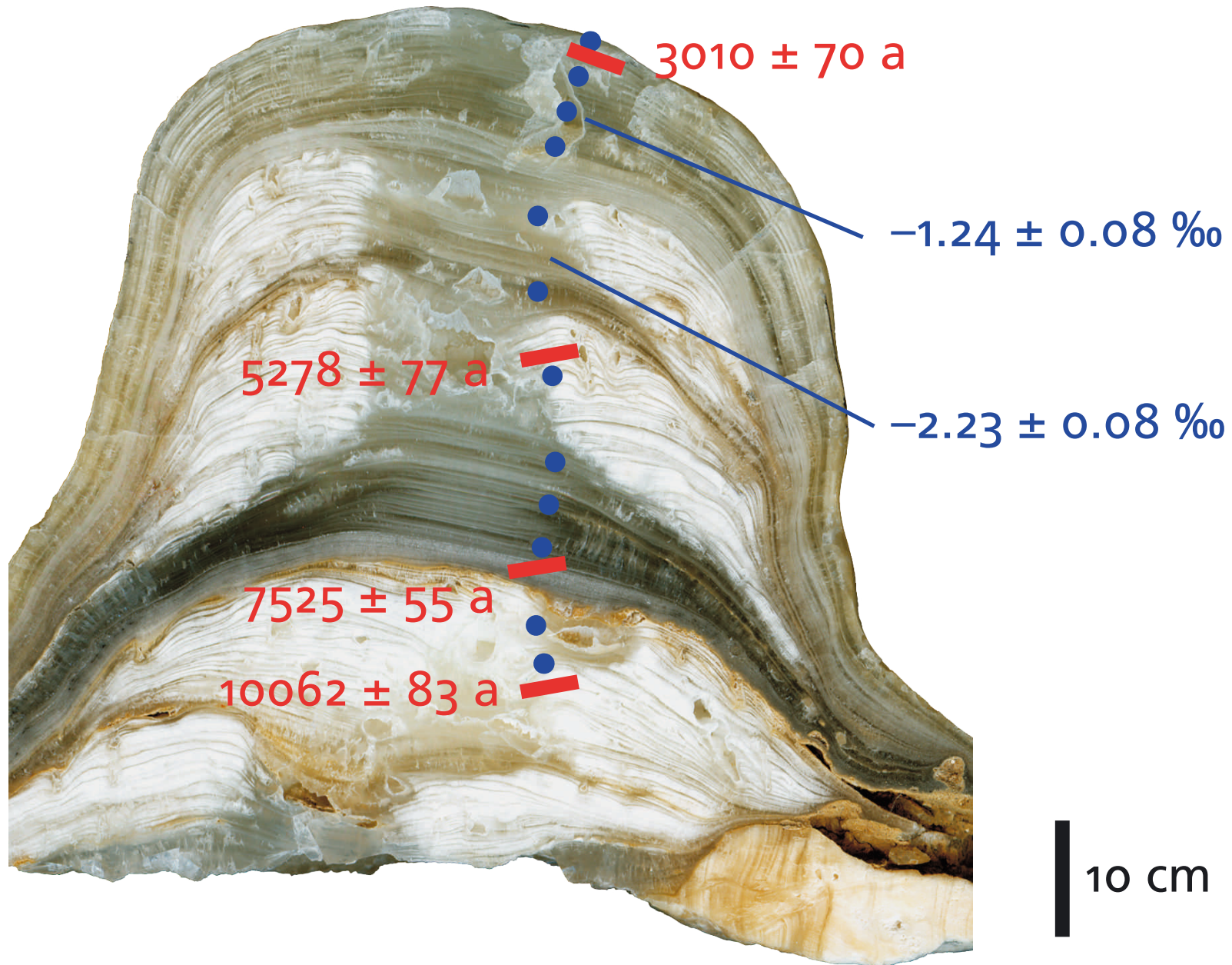


Paleoproxies: The Effects of Timescale Errors



Manfred Mudelsee

1 Archive



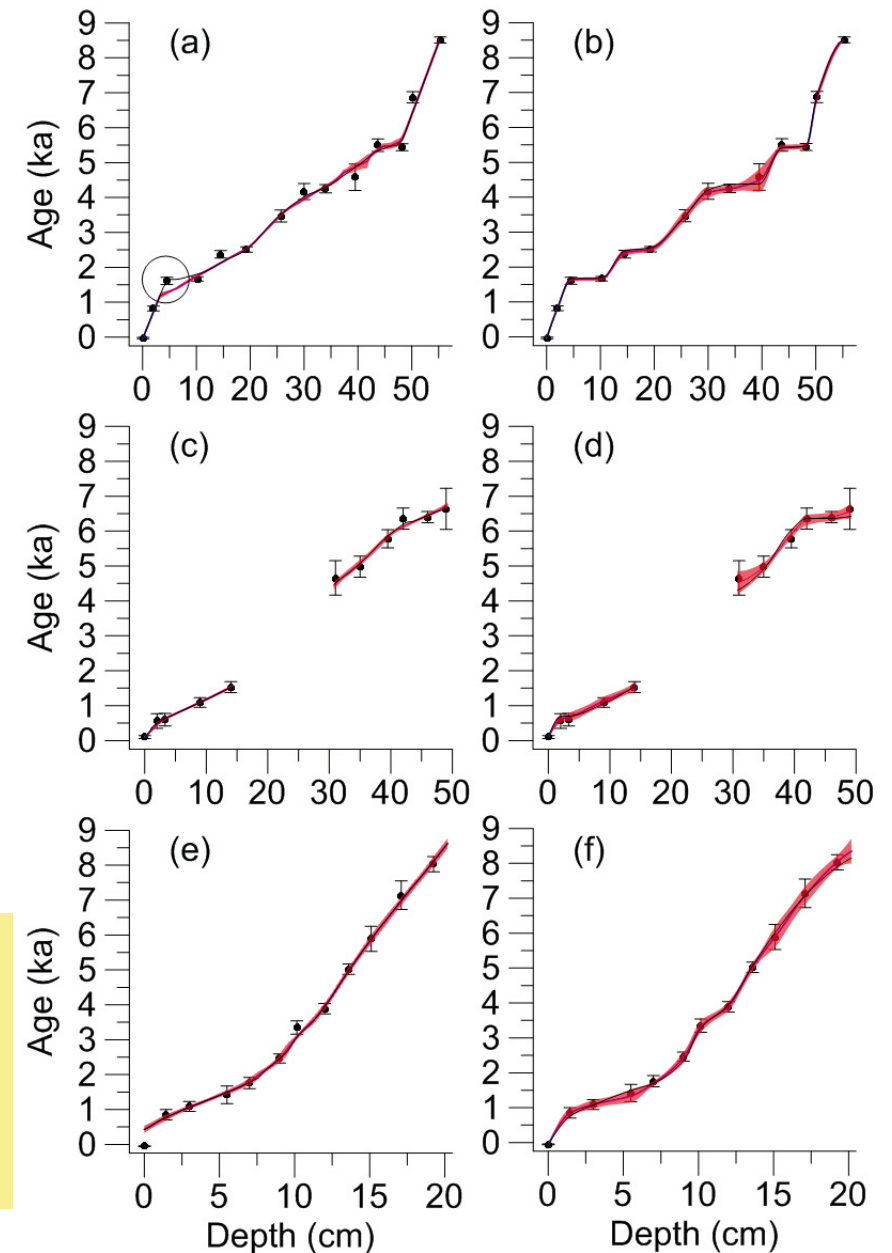
2 Timescale

Table 1. Dating points, stalagmite AH-1. Age-errors are given conventionally as 2- σ standard deviation.

Depth (cm)	Age (ka)	Age-error (ka)
0.1	-0.03	0.02
2	0.819	0.067
4.5	1.61	0.1
10.2	1.666	0.063
14.5	2.37	0.11
19.3	2.511	0.078
25.7	3.47	0.17
30	4.17	0.23
34	4.25	0.12
39.5	4.58	0.38
43.7	5.5	0.18
48.2	5.44	0.1
50.2	6.87	0.16
55.3	8.51	0.087

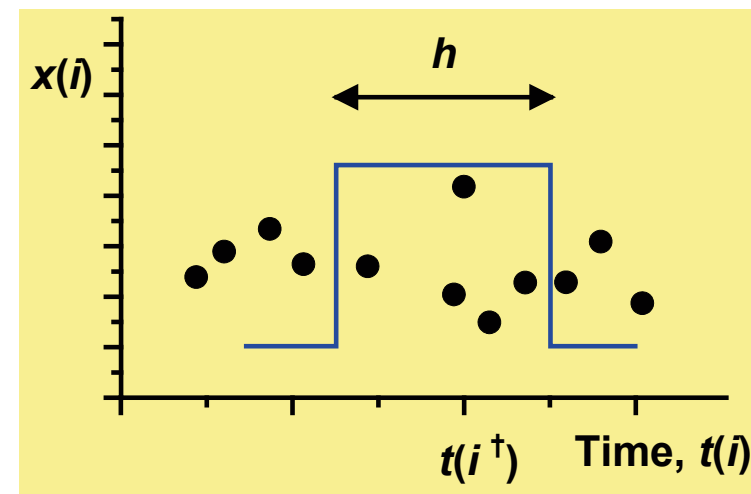
2 Timescale

Fig. 1. Age-depth curves for stalagmites AH-1 (**a, b**), Bu1 (**c, d**) and Bu4 (**e, f**) constructed with algorithms StalAge (**a, c, e**; Scholz and Hoffmann, 2011) and iscam (**b, d, f**; Fohlmeister, 2012). Shown are the dating points (black dots) with the 2- σ dating errors (vertical black bars), the constructed age-depth curves (thin black line) and the average from 2000 simulated age-depth curves (thick blue line – legible only in the zoomed electronic paper) with the corresponding 2- σ standard-error band (red shading). The circle in (**a**) indicates a systematic shift of simulated ages, which results from the methods used for obtaining the simulated curves (see Sects. 2.2.1 and 4.2).



3 Estimation Problem: Nonparametric Regression or Smoothing

Running mean



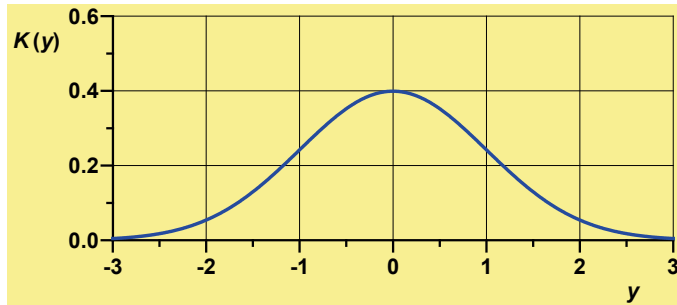
$$\hat{X}_{\text{trend}}^{\text{PC}}(T) = h^{-1} \sum_{i=1}^n [T(i) - T(i-1)] K \left[\frac{T - T(i)}{h} \right] X(i), \quad (4.52)$$

Kernel smoothing

K , kernel function ($K = 0$ outside, $K = \text{const.} > 0$ inside)

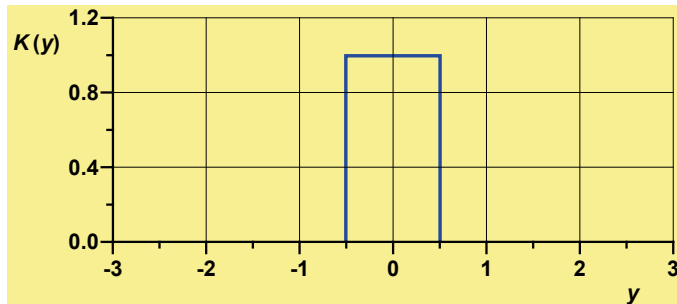
h , bandwidth

3 Estimation Problem: Nonparametric Regression or Smoothing



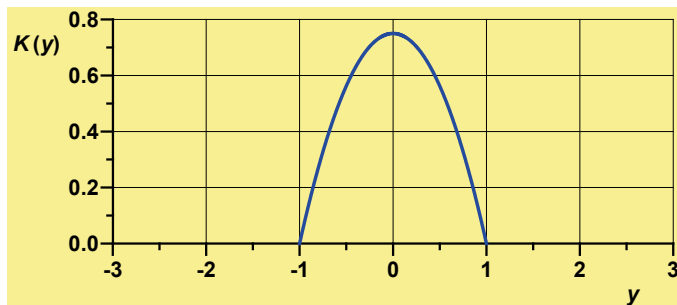
$$K(y) = (2\pi)^{-1/2} \exp(-y^2/2)$$

Gaussian kernel



$$K(y) = 1 \text{ for } |y| \leq 1/2$$

Uniform kernel



$$K(y) = 0.75 (1 - y^2) \text{ for } |y| \leq 1$$

Epanechnikov kernel

3 Estimation Problem: Nonparametric Regression or Smoothing

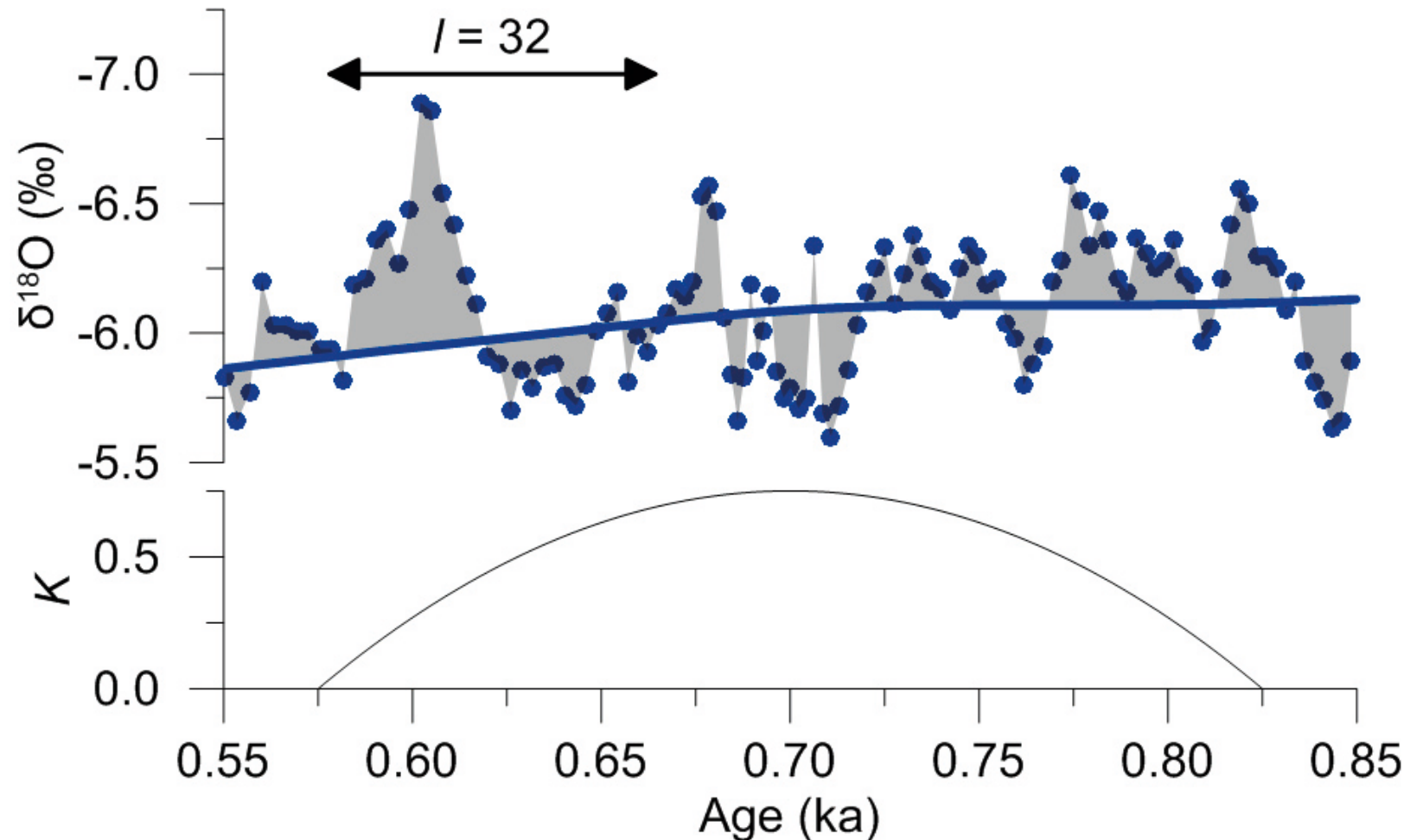
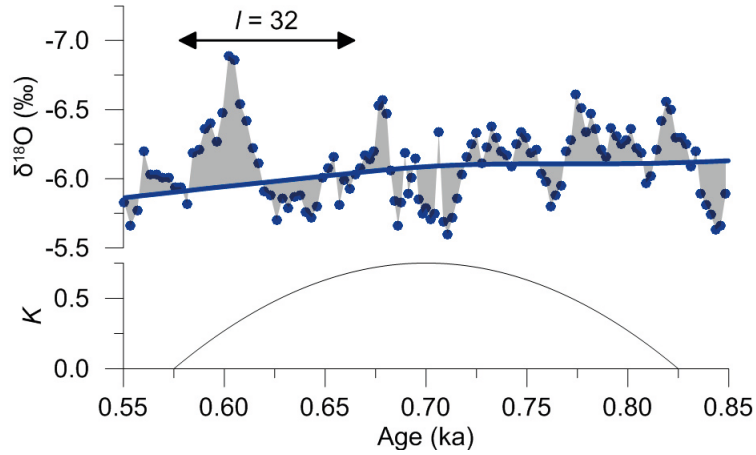


Fig. 2. Kernel trend estimation and MBB resampling illustrated by means of the Bu1 $\delta^{18}\text{O}$ record (blue dots, upper panel) during the interval from 0.55 to 0.85 ka on the StalAge-derived timescale. Also shown is trend estimate (blue line), regression residuals (indicated by the shading), parabolic kernel function K with a bandwidth of $h = 250$ a (black line, lower panel) and one block of length $l = 32$ for MBB resampling.

4 Standard Error Band



- 1 Data $\{t(i), x(i)\}_{i=1}^n$
- 2 Nonparametric regression (Eq. 3),
 inserted trend value for time $t(i)$

$$\hat{x}_{\text{trend}}(i) = h^{-1} \sum_{i=1}^n \left[\int_{s(i-1)}^{s(i)} K \left(\frac{t(i)-y}{h} \right) dy \right] x(i),$$

$$i = 1, \dots, n$$
- 3 Regression residuals

$$e(i) = x(i) - \hat{x}_{\text{trend}}(i), \quad i = 1, \dots, n$$
- 4 Apply MBB (Fig. 2) to $\{e(i)\}_{i=1}^n$

$$\{e^*(i)\}_{i=1}^n$$
- 5 Resampled $x(i)$

$$x^*(i) = \hat{x}_{\text{trend}}(i) + e^*(i), \quad i = 1, \dots, n$$
- 6 Simulated timescale (Section 2.2)

$$t^*(i), \quad i = 1, \dots, n$$
- 7 Resample (b , counter)

$$\{t^{*b}(i), x^{*b}(i)\}_{i=1}^n$$
- 8 Replication (continuous time, T)

$$\hat{x}_{\text{trend}}^{*b}(T)$$
- 9 Go to 4 until $b = B$ replications exist

$$\{\hat{x}_{\text{trend}}^{*b}(T)\}_{b=1}^B$$
- 10 Standard error given by
 standard deviation over B replications

$$\left\{ B^{-1} \sum_{b=1}^B \left[\hat{x}_{\text{trend}}^{*b}(T) - B^{-1} \sum_{b'=1}^B \hat{x}_{\text{trend}}^{*b'}(T) \right]^2 \right\}^{1/2}$$
- 11 Standard error-band given by
 trend estimate (continuous time, T)

$$\pm \text{standard error}$$

5 Results

$$h = 250 \text{ a}$$

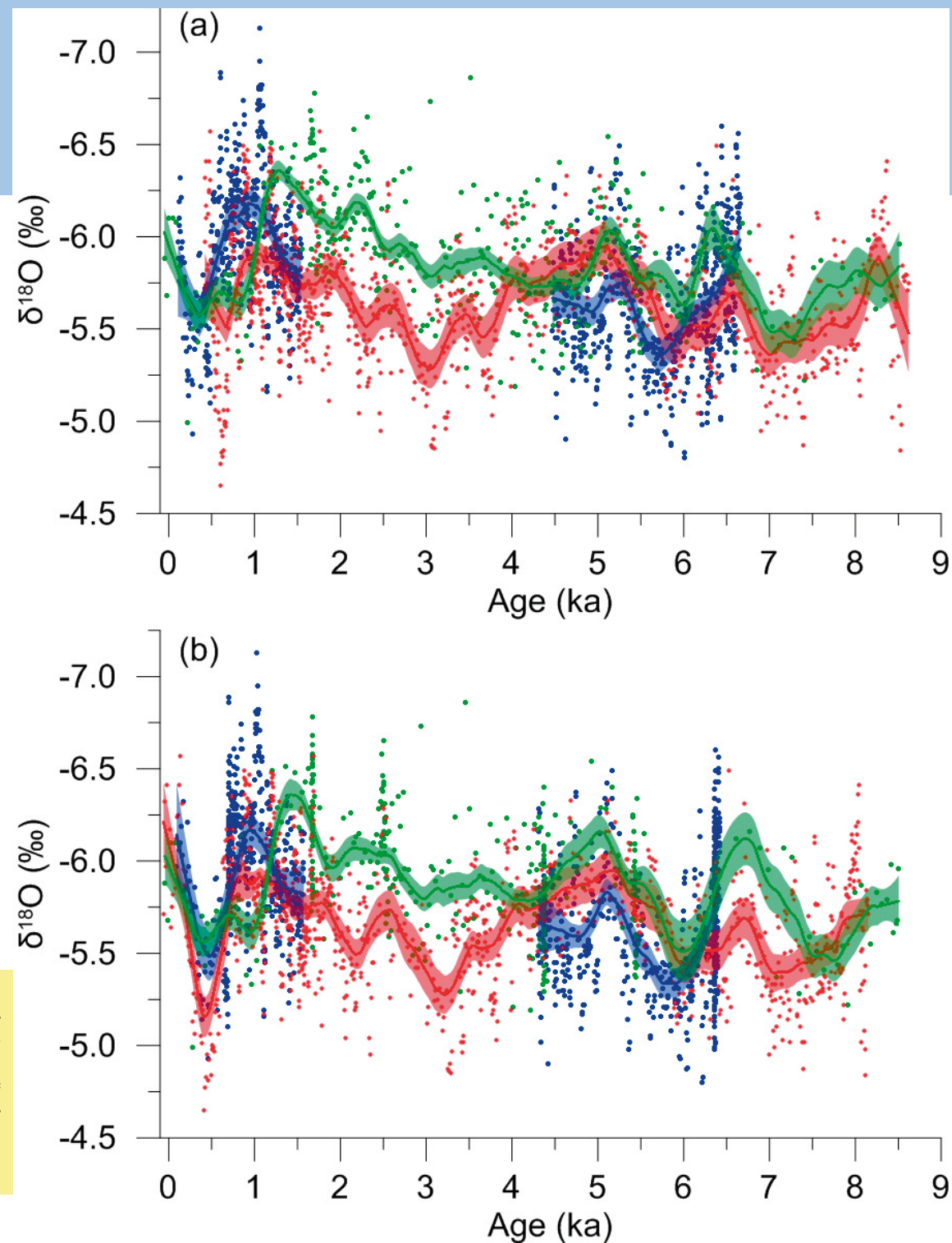
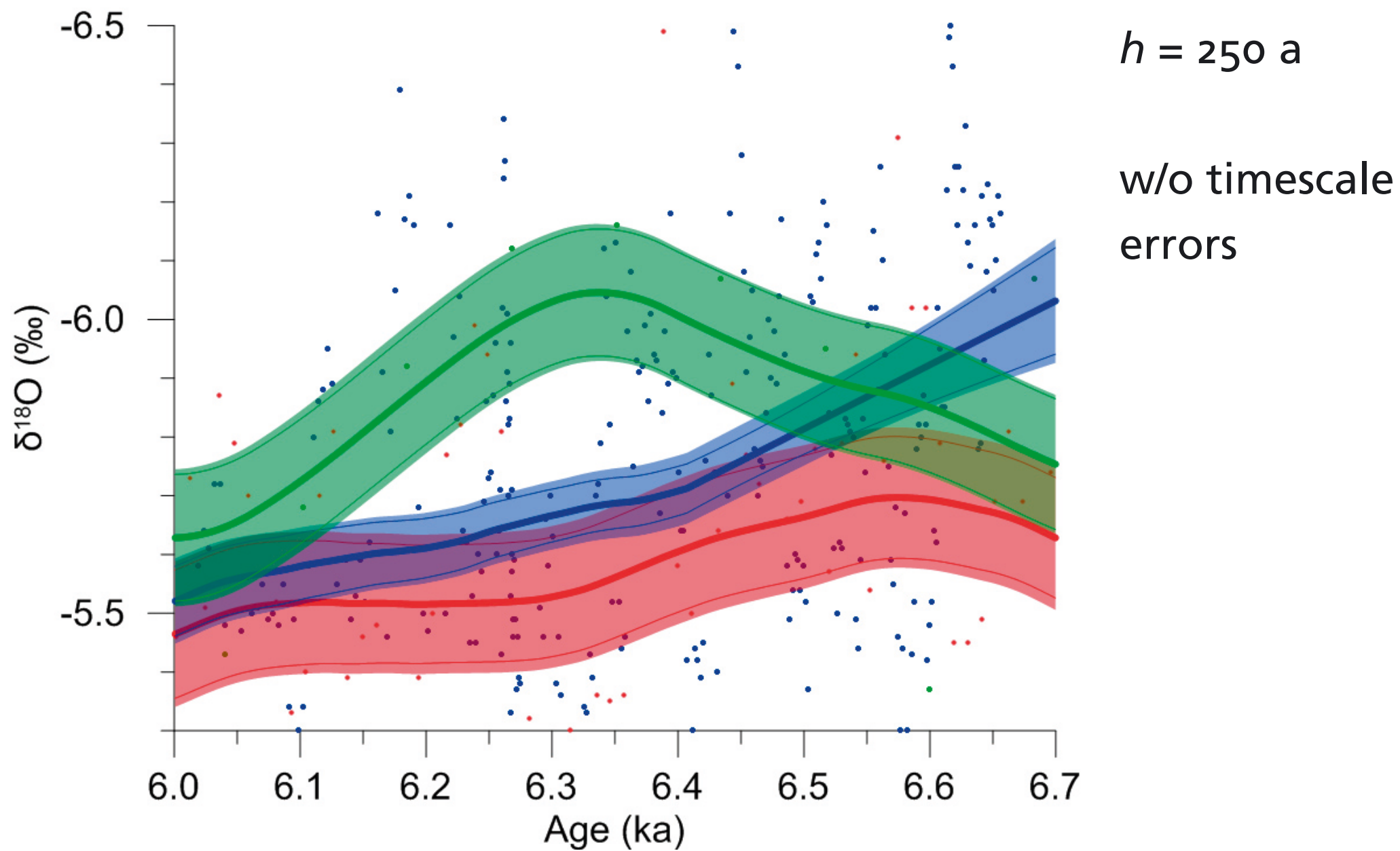
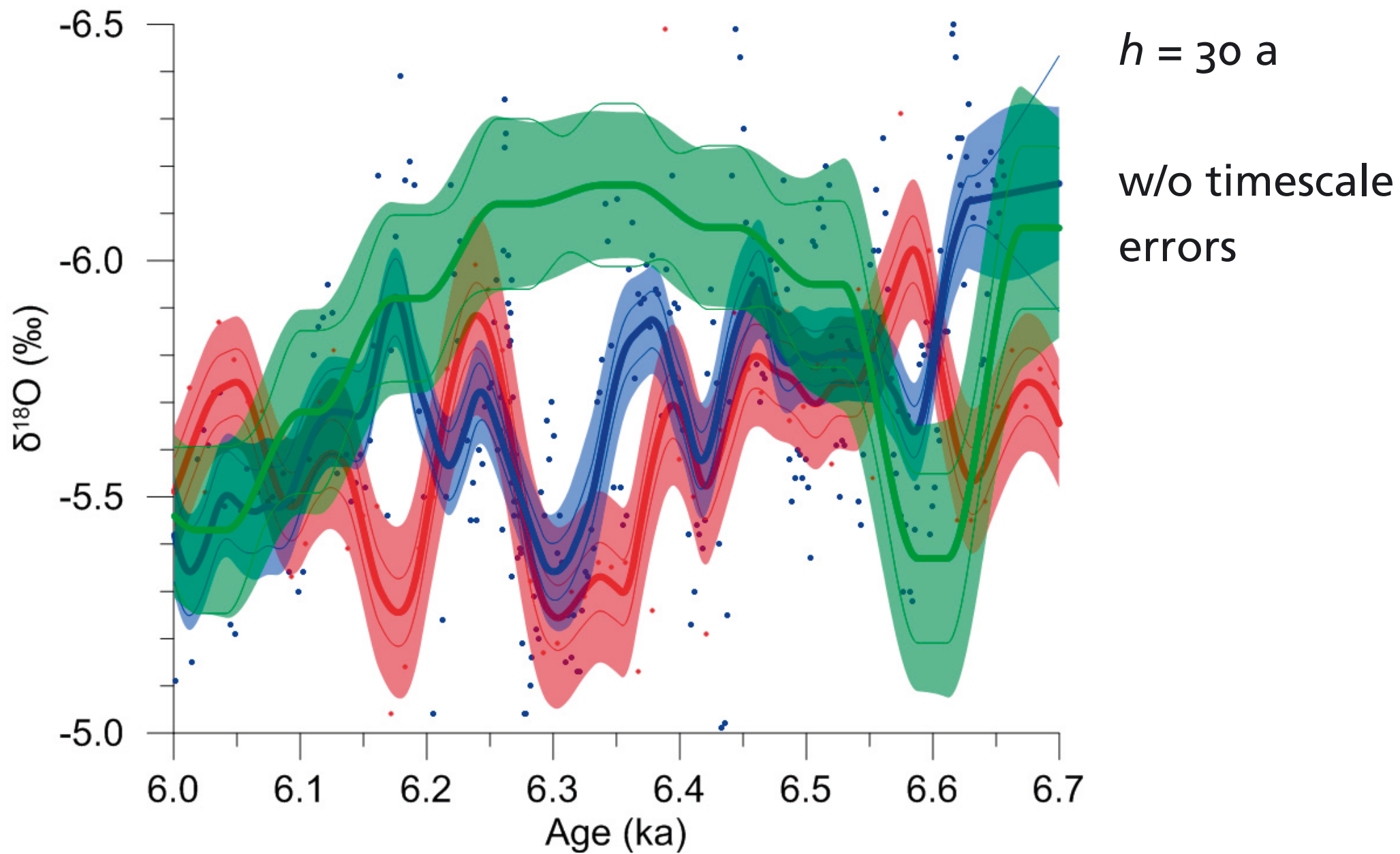


Fig. 4. Results, trend estimation on stalagmite time series for age-depth construction algorithms StalAge **(a)** and iscam **(b)**. Data points (dots), trends (thick solid lines) and 1- σ standard-error bands (shaded) are shown for stalagmites AH-1 (green), Bu1 (blue) and Bu4 (red). The early and late parts of the Bu1 series are analysed separately. The trends are calculated with a kernel bandwidth of $h = 250 \text{ a}$ and the standard-error bands with a number of MBB resamplings of $B = 2000$ and taking dating errors into account. The block lengths for MBB resampling are for **(a)**: $l = 1$ (AH-1), 33 (Bu1, late part), 32 (Bu1, early part) and 22 (Bu4); and for **(b)**: $l = 1$ (AH-1), 32 (Bu1, late part), 33 (Bu1, early part) and 22 (Bu4).

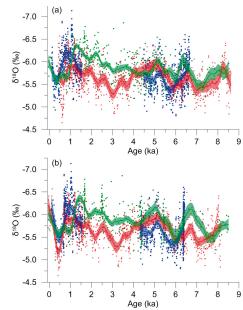
5 Results



5 Results

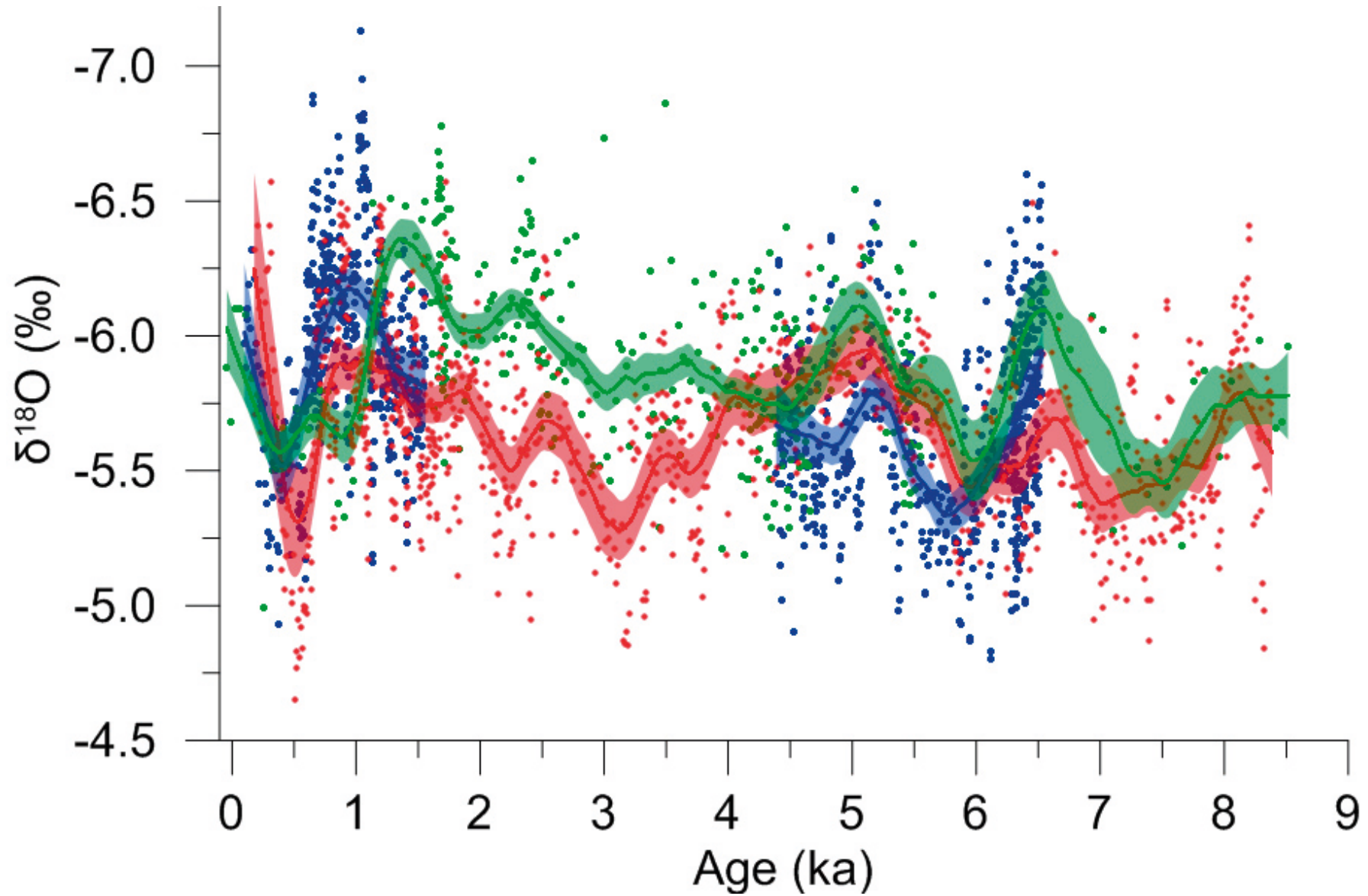


5 Results

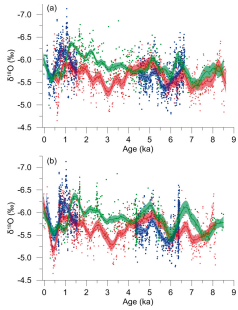


$h = 250$ a

model
uncertainty



5 Results



$h = 250$ a

$\delta^{18}\text{O}$ (‰)

model
uncertainty

Thanks! Enjoy your meal!

Thom answers your questions?

Talk is at www.manfredmudelsee.com