

RAMPFIT—a Fortran 77 program for ramp function regression

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Abstract

I propose a method for fitting a “ramp” to measured data. This is a continuous function, segmented in three parts: $x_{\text{fit}}(t) = x_1$ for $t \leq t_1$, x_2 for $t \geq t_2$, and linearly connected between t_1 and t_2 . Its purpose is to measure transitions in the mean of time series as they occur, for example, in paleoclimatic records. The unknowns x_1 and x_2 are estimated by weighted least-squares regression, t_1 and t_2 by a brute-force search. Computing costs are reduced by several means. The presented Fortran 77 program, RAMPFIT, includes analysis of weighted ordinary residuals for checking the validity of the ramp form and other assumptions. It fits an AR(1) model to the residuals to measure serial dependency; uneven time spacing is thereby allowed. Three bootstrap resampling schemes (parametric, nonparametric stationary, and wild) provide uncertainties for the estimated parameters. RAMPFIT works interactively (calculation/visualization). Example time series (1 artificial, 3 measured) demonstrate that this approach is useful for practical applications in geosciences (n less than a few hundred, noise, unevenly spaced times), and that the ramp function may serve well to model climate transitions.

Key Words: Time series analysis, Nonlinear regression, Three-phase regression, O-18/O-16, Sr-87/Sr-86, Neogene, Pleistocene.

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Preface

This report deals with a somewhat neglected model in regression analysis: the ramp (Figure 1). I argue that, in these days of computer power, this is a helpful function for investigating essentially gradual transitions in the mean of a time series, as they occur, for example, in paleoclimatic records.

Besides interested statisticians, it is my intention that the readership will include also open-minded paleoclimatologists. The argument of being too complex does not count: To follow, you merely require basic statistical concepts such as “mean” or “standard deviation”. You are invited to request a copy of the accompanying computer program, RAMPFIT.

Development of the idea and the program started at the end of 1994 during my dissertation studies at the Geological-Paleontological Institute, University of Kiel, FRG. It is a pleasure to thank my supervisor, Prof. Dr. Karl Stattegger, for his encouragement. Thanks go also to my co-supervisor, Prof. Dr. Michael Sarnthein. I thank Prof. Dr. Pieter Grootes (Leibniz Laboratory, University of Kiel, FRG), Dr. Michael Schulz (SFB 313, University of Kiel, FRG), Dr. Cees G. H. Diks (Department of Economics, University of Amsterdam, The Netherlands), Dr. Martin H. Trauth (Institute of Geosciences, University of Potsdam, FRG), and Dr. Olaf Müller (SAP-AG, Walldorf, FRG) for discussions. Here at the IMS, I thank Dr. Qiwei Yao and Alistair Duncombe for their reading of the manuscript and comments from the statistical perspective. However, I claim responsibility for any error left in the text.

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Questions, comments, and criticism are welcome! (My e-mail address: M.Mudelsee@ukc.ac.uk.)

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3 March 1999

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1 Introduction

Accurately quantifying past transitions is one prerequisite for a better causal understanding of climatic change. Given a time series $x(i)$ measured at times $t(i), i = 1, \dots, n$, which documents a transition in the mean, it is important to know: When did the transition start? When did it end? What were the levels of the mean before and after the transition? These questions lead ultimately to the three-phase regression model which is shown in Figure 1. We denote it a “ramp function.”

As physical motivation for the ramp form, consider a (climate) system at equilibrium which is disturbed by some external action over a period of time, then attaining a new equilibrium state.

The mathematical problem is to fit the ramp function,

$$x_{\text{fit}}(t) = \begin{cases} x_1, & \text{for } t \leq t_1, \\ x_1 + (t - t_1)(x_2 - x_1)/(t_2 - t_1), & \text{for } t_1 \leq t \leq t_2, \\ x_2, & \text{for } t \geq t_2, \end{cases}$$

to $x(i)$; that is, to estimate the parameters t_1 , x_1 , t_2 , and x_2 for the regression model

$$x(i) = x_{\text{fit}}(i) + \epsilon(i) \tag{1}$$

where $x_{\text{fit}}(i) := x_{\text{fit}}(t(i))$ is an abbreviation for the discrete form. We assume $\epsilon(i)$ to be distributed as $N(0, \sigma(i)^2)$, since for climate time series we anticipate heteroscedasticity. We further allow positive AR(1) dependency between the $\epsilon(i)$ since persistence is also typical for such time series. The two points of nondifferentiability (Figure 1) cause the main difficulty, in that they prevent us from formulating the normal equations for a least-squares (LS) estimation of t_1 and t_2 .

The proposed estimation method and the respective Fortran 77 program RAMPFIT are a pragmatic approach to that regression problem. By a brute-force search over a pre-defined grid, we find \hat{t}_1 and \hat{t}_2 , whereas we calculate \hat{x}_1 and \hat{x}_2 directly by using a weighted LS criterion.

The computational burden for that approach is reduced by several means. First, interactive graphics (for smoothing and time interval evaluation) allow us to keep the boundaries of the search grid narrow. Second, an intelligent search path on the grid permits updating of values of constants rather than new evaluation. Third, the search grid is as coarse as the time values $t(i)$ are.

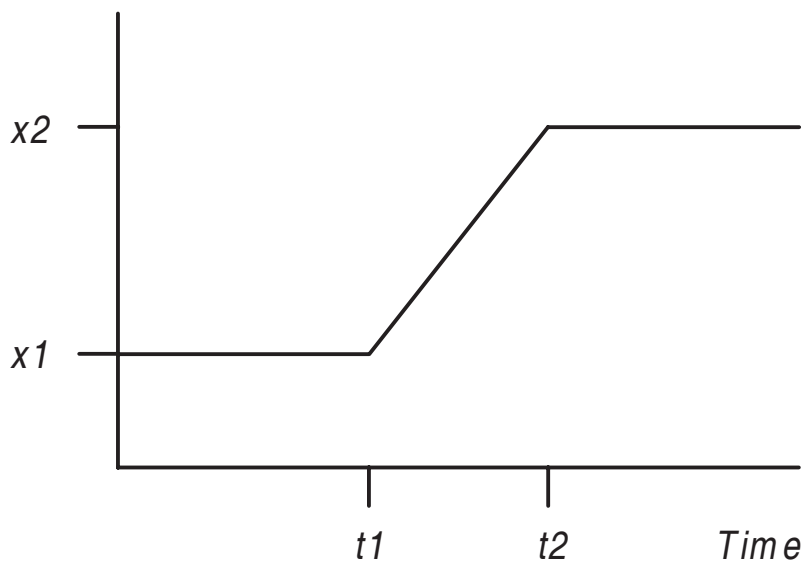


Figure 1: The ramp function regression model.

Section 2 explains the estimation method and also mentions possible extensions and alternatives. Section 3 demonstrates the analysis of the regression residuals which helps to assess whether the ramp function is indeed suited to the investigated data. Section 4 supplies three bootstrap resampling schemes for determining the uncertainties of the estimated parameters. Whereas an artificial time series serves to exemplify Sections 3 and 4, examples of measured data (Section 5) make clear that ramp function regression is of practical meaning for many types of time series encountered in the geosciences. The Appendix includes additional material (results, figures) and a short manual of RAMPFIT.

2 Ramp Function Estimation

2.1 Weighted LS

Since the standard deviation $\sigma(i)$ can vary with time, we use a LS criterion which puts heavier weights on values with smaller $\sigma(i)$. The best ramp

function fit to given data $x(i)$ thus minimizes

$$SSQW(t1, x1, t2, x2) = \sum_{i=1}^n [x(i) - x_{\text{fit}}(i)]^2 / \sigma(i)^2.$$

We may either know $\sigma(i)$ *a priori*, or estimate it per eye with the help of smoothing (Sections 2.3 and 3.3). We seek the solution $(\hat{t}1, \hat{x}1, \hat{t}2, \hat{x}2)$ for which $SSQW(\hat{t}1, \hat{x}1, \hat{t}2, \hat{x}2)$ is a minimum.

First, consider the case where $t1_{\text{search}} = t(i1)$ and $t2_{\text{search}} = t(i2)$ are fixed. Then we find $\hat{x}1$ and $\hat{x}2$ by putting

$$\partial/\partial x1 [SSQW(t1_{\text{search}}, x1, t2_{\text{search}}, x2)] \stackrel{!}{=} 0$$

and

$$\partial/\partial x2 [SSQW(t1_{\text{search}}, x1, t2_{\text{search}}, x2)] \stackrel{!}{=} 0.$$

This yields

$$\hat{x}2 = \left(\frac{K_3 K_4}{K_1} + K_6 \right) / \left(\frac{K_2 K_4}{K_1} + K_5 \right) \quad (2)$$

and

$$\hat{x}1 = (K_3 - \hat{x}2 K_2) / K_1. \quad (3)$$

The constants are:

$$K_1 = k_2 + (t1_{\text{search}} k_4 - k_5) / (t2_{\text{search}} - t1_{\text{search}}),$$

$$K_2 = k_3 - (t1_{\text{search}} k_4 - k_5) / (t2_{\text{search}} - t1_{\text{search}}),$$

$$K_3 = k_8,$$

$$K_4 = k_1 + [t2_{\text{search}}(t1_{\text{search}} + t2_{\text{search}})k_4 + 2 k_6 - (t1_{\text{search}} + 3 t2_{\text{search}})k_5] / (t2_{\text{search}} - t1_{\text{search}})^2,$$

$$K_5 = k_3 + [t1_{\text{search}}(t1_{\text{search}} + t2_{\text{search}})k_4 + 2 k_6 - (3 t1_{\text{search}} + t2_{\text{search}})k_5] / (t2_{\text{search}} - t1_{\text{search}})^2,$$

$$K_6 = k_9 - k_7 - 2 [t1_{\text{search}}k_{10} - k_{11}] / (t2_{\text{search}} - t1_{\text{search}}),$$

and

$$k_1 = \sum_{i=1}^{i1} 1/\sigma(i)^2,$$

$$k_2 = \sum_{i=1}^{i2-1} 1/\sigma(i)^2,$$

$$\begin{aligned}
k_3 &= \sum_{i=i_2}^n 1/\sigma(i)^2, \\
k_4 &= \sum_{i=i_1+1}^{i_2-1} 1/\sigma(i)^2, \\
k_5 &= \sum_{i=i_1+1}^{i_2-1} t(i)/\sigma(i)^2, \\
k_6 &= \sum_{i=i_1+1}^{i_2-1} t(i)^2/\sigma(i)^2, \\
k_7 &= \sum_{i=1}^{i_2-1} x(i)/\sigma(i)^2, \\
k_8 &= \sum_{i=1}^n x(i)/\sigma(i)^2, \\
k_9 &= \sum_{i=i_2}^n x(i)/\sigma(i)^2, \\
k_{10} &= \sum_{i=i_1+1}^{i_2-1} x(i)/\sigma(i)^2, \\
k_{11} &= \sum_{i=i_1+1}^{i_2-1} x(i) t(i)/\sigma(i)^2.
\end{aligned}$$

2.2 Brute-Force Search

Since the ramp function (Fig. 1) is not differentiable with respect to time at t_1 and t_2 , we cannot derive \hat{t}_1 and \hat{t}_2 in the same manner as \hat{x}_1 and \hat{x}_2 . Instead, we search over all points of a t_1 – t_2 grid (Fig. 2). The knots of this grid are determined by the time values $t(i)$ and the restriction $t_{1\text{search}} < t_{2\text{search}}$. For every search point, we calculate \hat{x}_1 , \hat{x}_2 , and $SSQW$.

The search starts in the upper left corner of the grid (Fig. 2) where constants k_1 – k_{11} have to be calculated explicitly. An intelligent path then permits updating of the values of the constants (Table 1). This reduces the computing costs by a factor of about 2.

We prescribe the boundaries of the t_1 – t_2 search grid. They have to be broad enough to reduce the possibility of a solution on the grid boundary. In such a case it would be likely that a better fit solution exists outside the grid. On the other hand, the grid boundaries have to be narrow enough to

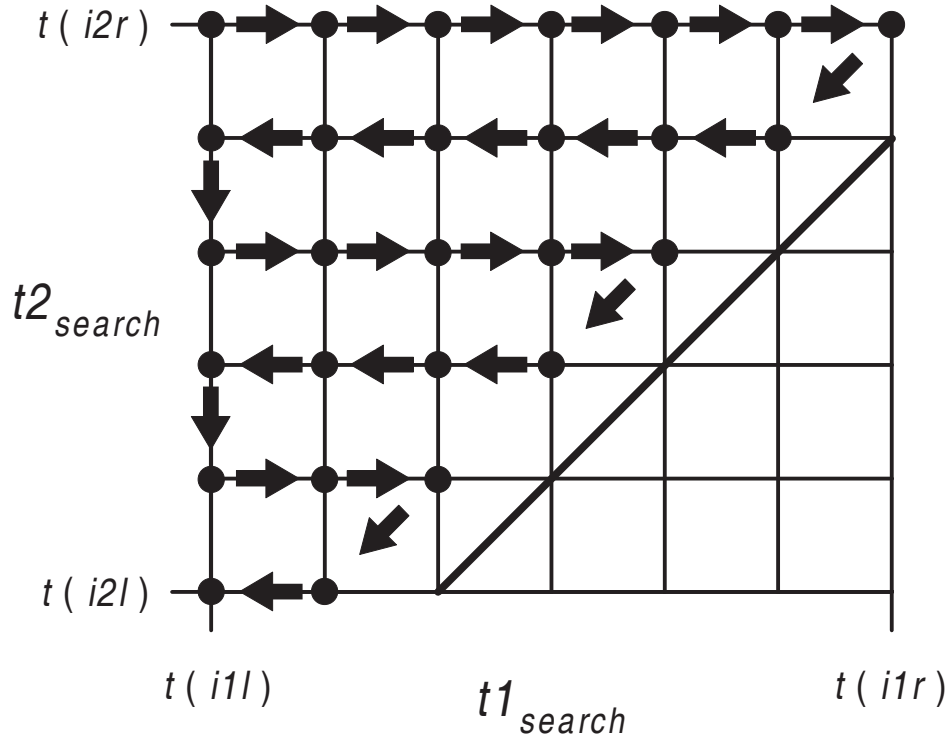


Figure 2: Search grid for \hat{t}_1 and \hat{t}_2 : search points (\bullet), search steps (arrows) and 1:1 line (heavy).

keep computing costs small. The computer program RAMPFIT, described in Appendix B, helps to suit the search boundaries by means of interactive graphics (smoothing).

The coarseness of the search grid, that is, the spacing of the $t(i)$, limits the precision of \hat{t}_1 and \hat{t}_2 . However, when the uncertainties of \hat{t}_1 and \hat{t}_2 (which are determined by bootstrap resampling, Section 4) are larger, that limitation becomes unimportant. The brute-force search, despite its computational burden and its tedious appearance, has the advantage of providing a *global* optimal solution within the search grid.

2.3 Standard Deviation

The time-dependent standard deviation provides the weights for the LS regression. If no prior knowledge (e. g., measurement error) is available, we

Table 1. Updating of constants k_1 – k_{11} at a point ($t1 = t(i1)$, $t2 = t(i2)$) on the search path (Fig. 2). The arrows indicate possible directions of the last step. For brevity, we write $w(i) := 1/\sigma(i)^2$

Updating		←	↓	↘
	→			
k_1	$+w(i1)$	$-w(i1+1)$		$-w(i1+1)$
k_2			$-w(i2)$	$-w(i2)$
k_3			$+w(i2)$	$+w(i2)$
k_4	$-w(i1)$	$+w(i1+1)$	$-w(i2)$	$+w(i1+1) - w(i2)$
k_5	$-t(i1)w(i1)$	$+t(i1+1)w(i1+1)$	$-t(i2)w(i2)$	$+t(i1+1)w(i1+1) - t(i2)w(i2)$
k_6	$-t(i1)^2w(i1)$	$+t(i1+1)^2w(i1+1)$	$-t(i2)^2w(i2)$	$+t(i1+1)^2w(i1+1) - t(i2)^2w(i2)$
k_7			$-x(i2)w(i2)$	$-x(i2)w(i2)$
k_9			$+x(i2)w(i2)$	$+x(i2)w(i2)$
k_{10}	$-x(i1)w(i1)$	$+x(i1+1)w(i1+1)$	$-x(i2)w(i2)$	$+x(i1+1)w(i1+1) - x(i2)w(i2)$
k_{11}	$-x(i1)t(i1)w(i1)$	$+x(i1+1)t(i1+1)w(i1+1)$	$-x(i2)t(i2)w(i2)$	$+x(i1+1)t(i1+1)w(i1+1) - x(i2)t(i2)w(i2)$

have to estimate $\sigma(i)$. We assume also $\sigma(i)$ to have a ramp form. This is motivated by the definition of climate not only in terms of mean value but also in terms of variability. A climate change therefore may consist of a ramp function change of standard deviation.

RAMPFIT allows us to fit $\sigma(i)$ per eye. Within a running window, the standard deviation of the data is calculated. Interactive graphics and an adjustable degree of smoothing facilitate the estimation (Sections 3.3 and B.2).

In principle, it would be possible to carry out a LS regression of the standard deviation or to use more complicated functions for $\sigma(i)$. Since the major interest in the present context is an appropriate distribution of the weights, we avoid such extensions.

2.4 Extensions and Alternative Approaches

My approach of brute-force search is similar to Williams' (1970) adaption of Hudson's search to a three-phase linear regression. He used a search path which made re-calculation of constants necessary, something our path (Fig. 2) completely avoids. He further conducted a "fine estimation" to overcome the coarseness of the grid.

One possible alternative to brute-force search is numerical minimization of $SSQW(t1, x1, t2, x2)$ which also would provide a "fine estimation." A numerical technique without derivative calculation, such as Brent's search in two dimensions, would be necessary. The solution, however, would only be a local minimum. Typical questions such as choice of starting point and stopping rule would arise. Furthermore, updating of constants which reduces computing costs would not be possible.

Since we allow AR(1) dependency between the errors $\epsilon(i)$, generalized LS estimation could be a useful extension, especially for a strong dependency. In addition to $\sigma(i)$, the serial correlation would have to be determined. Note that because of the bootstrap resampling (Section 4), correlated errors do not distort evaluated uncertainty ranges of estimated parameters.

We consider robust regression to be a less helpful extension. The simple calculation of $\hat{x}1$ and $\hat{x}2$ by LS minimization (Equations 2 and 3) would not be possible. Most climate time series seem not to depart so strongly from the Gaussian shape that would justify such a complication.

The statistical literature to "change-point detection" indicates alternative approaches to ramp function regression. For example, we could use tests for an abrupt change in the mean (see, e. g., Basseville and Nikiforov, 1993) to

estimate $t1$ “from the left” (Figure 1) and $t2$ “from the right.” However, when a significant number of time series points lie *between* $t1$ and $t2$, that approach loses considerable power.

It might be interesting to adapt the approach of Tishler and Zang (1981) and approximate $x_{\text{fit}}(t)$ by a continuously differentiable function in some arbitrarily small neighborhood of $t1$ and $t2$. That smooth regression model could then be estimated by numerical techniques with derivative calculation. Similar questions to above would arise, and updating would not be possible.

We finally mention change-point detection by nonparametric regression (Müller, 1992).

2.5 Computing Costs

Costs not only depend on the number of data points, n , and the number of grid search points (Figure 2), say n_{search} , but also on the proportion of data points between $t1$ and $t2$ because in that interval evaluation of constants is more expensive. The costs to calculate k_1-k_{11} for one search point are roughly proportional to n whereas updating costs for all search points are roughly proportional to n_{search} . Thus, for smaller values of n/n_{search} , numerical minimization which requires calculation only at a small number of search points, is an interesting alternative.

Costs further are determined by the type of machine and compiler. The following numbers are based on RAMPFIT test calculations with a PC with 80486-50 DX processor and the Microsoft Fortran PowerStation 1.0 compiler, and a Sunsparc workstation with 143 MHz clock and the EPCF77 2.7 compiler. They should be seen as a rough guideline.

For $n = 500$, the CPU time per search point is about 150 μs on the workstation, for $n = 300$ about 50 μs , and for $n = 100$ about 18 μs . The PC was about 100 times slower.

3 Residual Diagnostics

We have made the following assumptions for the ramp function regression:

1. The errors $\epsilon(i)$ are Gaussian distributed.
2. The errors have time-dependent standard deviation $\sigma(i)$, that is, the weighting is appropriate.

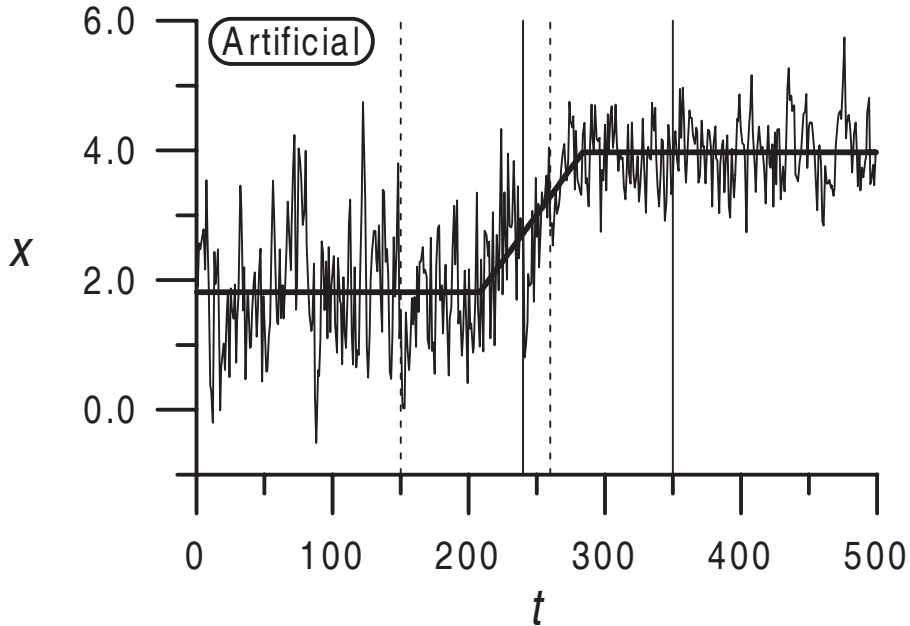


Figure 3: Ramp function fit (heavy line) to the artificial time series (Tables 2 and 3). Grid boundaries for the $t1$ -search (dashed vertical lines) and the $t2$ -search (solid).

3. The relationship between x and t is indeed a ramp function.
4. The errors show AR(1) dependency (that is not too strong).

Analysis of regression residuals is an important tool for assessing how well these assumptions are fulfilled. For an excellent treatment of that subject I refer to Chapter 3 of Montgomery and Peck (1992). In the present study, an artificial time series (Fig. 3, Tables 2 and 3) illustrates residual analysis.

3.1 Weighted Ordinary Residuals

The weighted ordinary residuals are calculated as

$$e(i) = [x(i) - \hat{x}_{\text{fit}}(i)] / \hat{\sigma}(i)$$

where $\hat{x}_{\text{fit}}(i)$ is the ramp function fitted by brute-force search and LS minimization, and $\hat{\sigma}(i)$ is the standard deviation fitted per eye. Given the un-

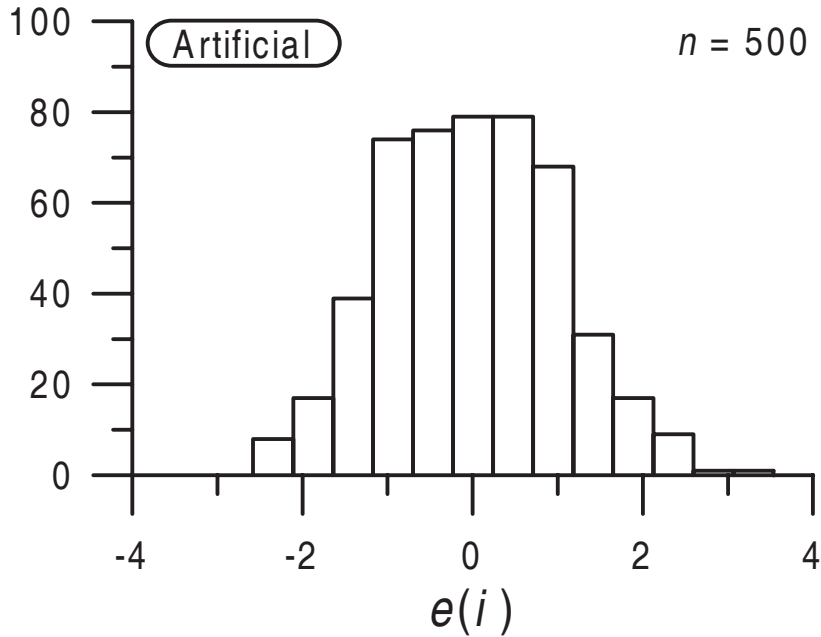


Figure 4: Histogram of the weighted ordinary regression residuals, artificial time series.

certainty of the latter fit, we regard it unnecessary to furthermore studentize $e(i)$.

3.2 Gaussian Shape

A histogram of the $e(i)$ (Figure 4) is the first visual tool for examining a Gaussian shape. We follow Scott (1979) and use the number of classes

$$NINT \left[(e_{\max} - e_{\min}) n^{1/3} / (3.49 e_{\text{std}}) \right]$$

where $NINT()$ is the nearest integer function; and e_{\max} , e_{\min} , and e_{std} are, respectively, maximum, minimum, and standard deviation of $e(i)$.

Figure 5 shows the normal probability plot, $\Phi^{-1}[(i-1/2)/n]$ vs. $e_{[i]}$ (where Φ^{-1} is the inverse cumulative normal function and $e_{[i]}$ are the ranked residuals); the Gaussian shape appears as a straight line, especially in the crucial central part of the plot. RAMPFIT uses the approximation of Hamaker (1978) to Φ^{-1} .

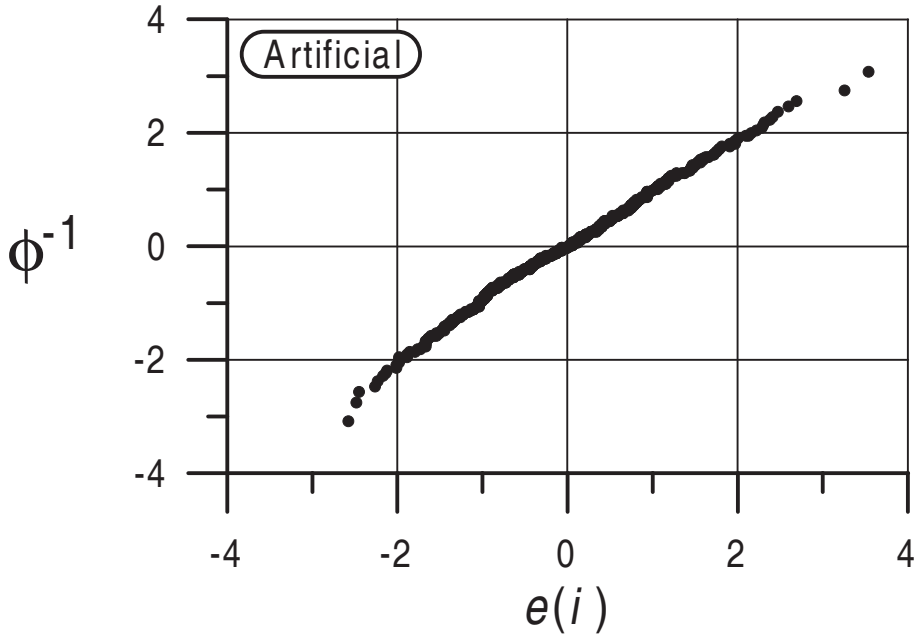


Figure 5: Normal probability plot of the weighted ordinary regression residuals, artificial time series.

A stronger violation of the Gaussian assumption would necessitate more robust measures than LS regression (Section 2.4).

3.3 Appropriate Weighting

In Figure 6, $\hat{\sigma}(i)$ (fitted per eye) is plotted together with the standard deviation of the detrended values, $[x(i) - x_{\text{fit}}(i)]$. Their standard deviation is calculated within a window which contains a fixed number of points and which is shifted on the time axis (k -nearest-neighbor smoothing). The assigned time value is the average of time values contained in the window (Tukey, 1977). That is more precise since, for climate time series, we expect uneven time spacing. RAMPFIT facilitates comparison with $\hat{\sigma}(i)$ by allowing us to choose the degree of smoothing. RAMPFIT also gives the value of reduced chi-square,

$$SSQWN = SSQW(t\hat{1}, x\hat{1}, t\hat{2}, x\hat{2}) / (n - 4),$$

which—in the case of independent $\epsilon(i)$ —should be around unity for an ade-

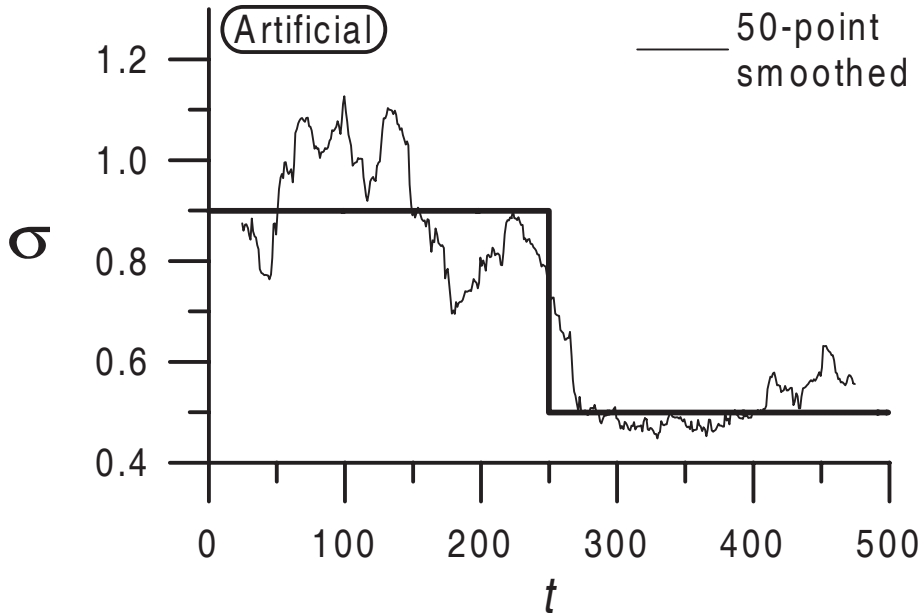


Figure 6: Per-eye fit (heavy line) of $\sigma(i)$ against standard deviation (smoothed) of the detrended values (light line), artificial time series. The smoothing window contains 50 points.

quate fit to $x(i)$ and a proper guess of $\sigma(i)$ (see, for example, Bevington and Robinson (1992: Chapter 11)). In the case of strong dependency, *SSQWN* is no longer useful, and, also, the smoothed standard deviation is then lower than the true. (The example of the artificial time series (Tables 2 and 3, Fig. 6) indicates that an autocorrelation coefficient of 0.4 is not a strong dependency in that respect.) Note that this underestimation would be a roughly *constant percentage* over time, provided the coefficient of variation of the time spacing is small. Thus, the underestimation would not corrupt the appropriate distribution of the weights.

3.4 Suitable Ramp Model

The plots of $e(i)$ vs. $t(i)$ (Fig. 7) and $e(i)$ vs. $x_{\text{fit}}(i)$ (Fig. 8), together with the fit (Fig. 3), help to identify outliers or systematic deviations from the assumed ramp form. In the case of the artificial time series with a pre-defined

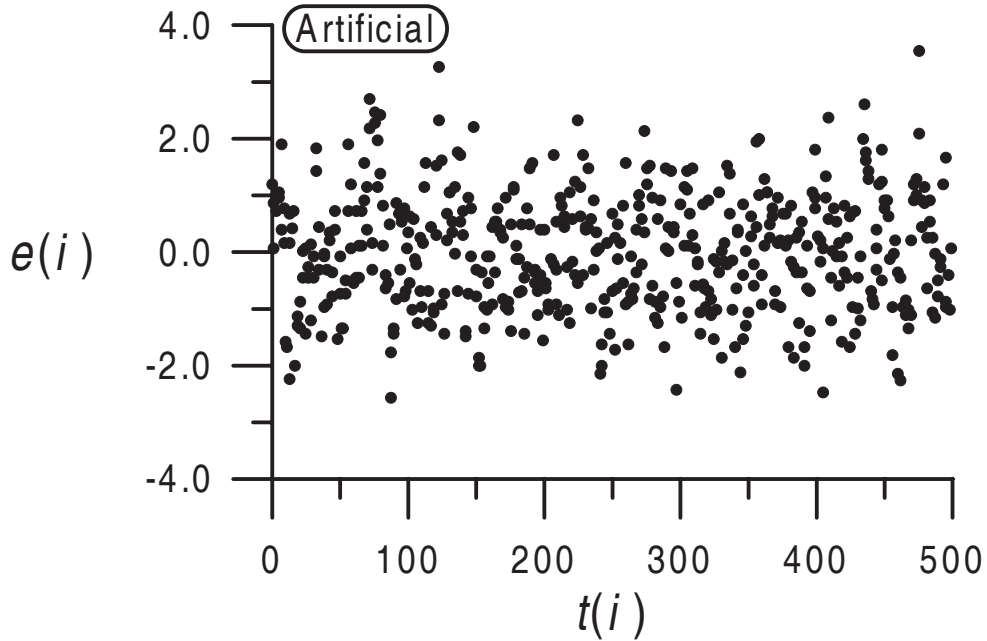


Figure 7: Weighted ordinary regression residuals vs. time, artificial time series.

ramp transition, as expected, we observe no such behavior: The points are homogeneously distributed.

3.5 Weak AR(1) Dependency

We inspect AR(1) dependency between errors by means of a lag-1 scatterplot of $e(i)$ (Fig. 9). The orientation of the cloud of points along the 1:1 line indicates a positive correlation. We quantify that dependency by fitting the stationary time series model

$$e(i) = a^{[t(i)-t(i-1)]}e(i-1) + [1 - a^{2[t(i)-t(i-1)]}]^{0.5}u(i) \quad (4)$$

to the residuals where the $u(i)$ are assumed to be independent and identically distributed as $N(0, 1)$. This model is a generalization of an AR(1) model to discrete data with an uneven time spacing, the parameter a corresponding to the autocorrelation coefficient. Estimation is carried out using a LS criterion and numerical techniques. RAMPFIT also gives the estimated value of $\tau =$

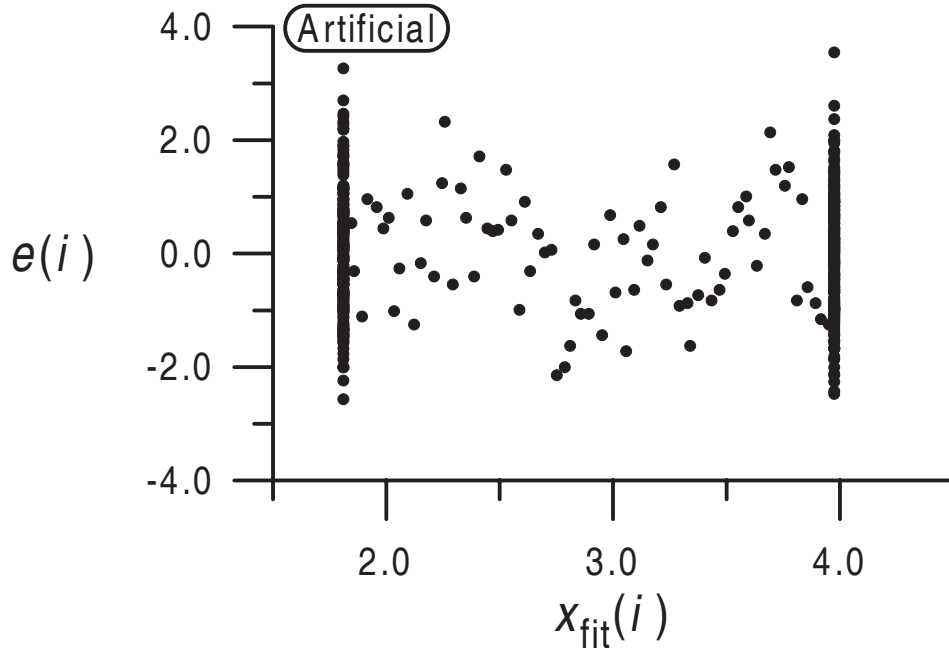


Figure 8: Weighted ordinary regression residuals vs. x_{fit} , artificial time series. (Because of the ramp form, clusters of points at \hat{t}_1 and \hat{t}_2 emerge.)

$-1/\ln(a)$ which is the decay period of the autocorrelation function of the AR(1) model. My approach is similar to, amongst others, Robinson's (1977).

In the case of the artificial time series, we estimate $\hat{a} = 0.409$ and $\hat{\tau} = 1.119$, that is, approximately the true serial correlation. It would be very difficult to evaluate the bias and the variance of estimator \hat{a} analytically. Despite that, \hat{a} serves well for judging whether the dependency between the errors is weak enough to circumvent a generalized LS estimation, and whether $SSQWN$ is a useful quantity (Section 3.3).

Strictly speaking, we do not test whether the dependency is indeed first-order autoregressive which, for unevenly spaced time series, would be a formidable task. This omission should be justified in case of climate time series ("natural persistence").

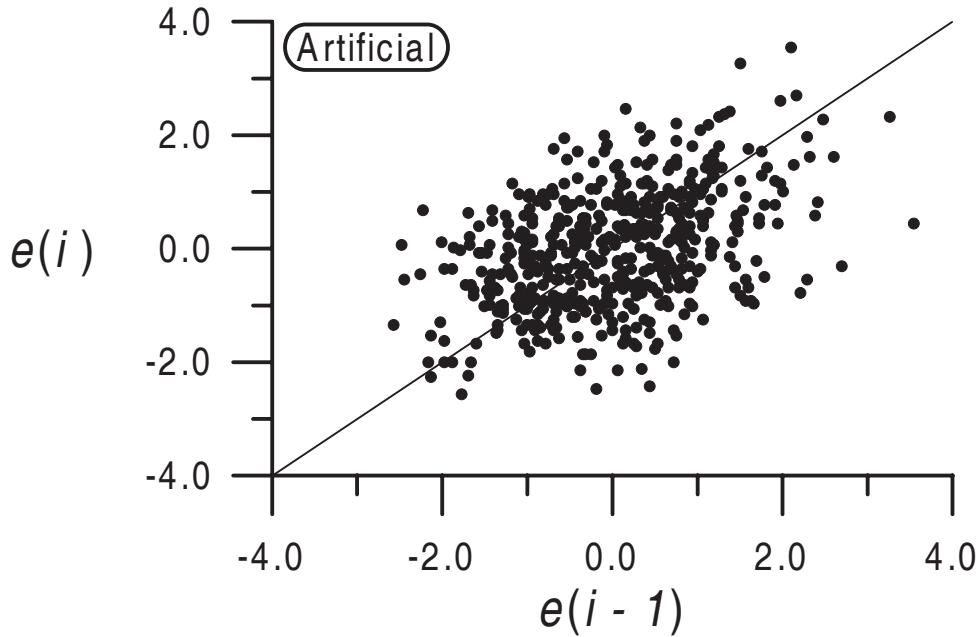


Figure 9: Lag-1 scatterplot of weighted ordinary regression residuals (artificial time series), 1:1 line.

4 Bootstrap Resampling

After we have fitted a ramp function and—assisted by the residual plots—accepted that model for a particular climate time series, geological interpretation of the fit result requires knowledge about how close the estimated parameters are to the true parameters. Those biases and uncertainties of $\hat{t}1$, $\hat{x}1$, $\hat{t}2$, and $\hat{x}2$, we estimate by bootstrap resampling: From the original time series $x(i)$, we draw a first sample $x(i)^{(j=1)}$ that is intended to have the same statistical properties; three methods of doing so are described in this Section. A ramp function is fitted to $x(i)^{(j=1)}$, resulting in $\hat{t}1^{*(j=1)}$, $\hat{x}1^{*(j=1)}$, $\hat{t}2^{*(j=1)}$, and $\hat{x}2^{*(j=1)}$. Resampling and fitting is repeated, B times in total. The distributions of $\hat{t}1^{*(j)}$, $\hat{x}1^{*(j)}$, $\hat{t}2^{*(j)}$, and $\hat{x}2^{*(j)}$ then inform about biases and uncertainties.

Following Efron and Tibshirani (1993) in their excellent monography on the bootstrap, we use $B = 200$. With regard to CPU time and coarseness of estimation ($t1-t2$ grid), a higher number of replications would make no

sense. The search grid (Fig. 2) remains unchanged.

The more traditional approach for estimating uncertainties, via the second derivative of $SSQW$, is not applicable since the ramp function is not continuously differentiable. The numerical minimization technique (Section 2.4), however, can use that approach.

4.1 Parametric Bootstrap

The parametric bootstrap resamples $x(i)^{*(j)}$ after Equation 1:

$$x(i)^* = \hat{x}_{\text{fit}}(i) + \hat{\sigma}(i) v(i), \quad i = 1, \dots, n,$$

where $\hat{x}_{\text{fit}}(i)$ is the estimated fit function, $\hat{\sigma}(i)$ is the estimated standard deviation, and the $v(i)$ are distributed as $N(0, 1)$ with the AR(1) correlation in Equation 4.

For generating the random numbers, RAMPFIT uses the algorithm of Park and Miller (1988) in combination with routine Gasdev from Press and others (1992).

In the histogram of $\hat{t}1^{*(j=1, \dots, B=200)}$ (Fig. 10), the fit value, $\hat{t}1$, is marked. Also the median, $med[\hat{t}1^{*(j)}]$, is drawn with error bars $\pm mad'[\hat{t}1^{*(j)}]$ where $mad' := 1.4826 mad$. The mad is the median of absolute distances to the median; a normal distribution has standard deviation $\simeq mad'$. For comparison with $\hat{t}1$, we prefer those robust measures which are also meaningful for heavier skewed distributions. Fig. 10 reveals that $\hat{t}1$ has negligible bias. Furthermore, $mad'[\hat{t}1^{*(j)}] = 11.064$ is clearly larger than the average spacing (0.999) of the time series within the $t1$ search region, justifying the coarse $t1$ - $t2$ grid. In addition to med and mad' , RAMPFIT also calculates the average, standard deviation, minimum, and maximum; the same also applies for $\hat{x}1^{*(j)}$, $\hat{t}2^{*(j)}$, and $\hat{x}2^{*(j)}$. The two other bootstrap schemes are dealt with analogously.

The scatterplot between $\hat{t}1^{*(j)}$ and $\hat{t}2^{*(j)}$ (Fig. 11) portrays the correlation between these two quantities. RAMPFIT can draw such plots for any pair from the four parameters.

4.2 Nonparametric Stationary Bootstrap

The nonparametric stationary bootstrap (Politis and Romano, 1994), adapted to heteroscedasticity, uses the set $\{e(k); k = 1, \dots, n\}$ of regression residuals for resampling:

$$x(i)^* = \hat{x}_{\text{fit}}(i) + \hat{\sigma}(i) e(i)^*, \quad i = 1, \dots, n,$$

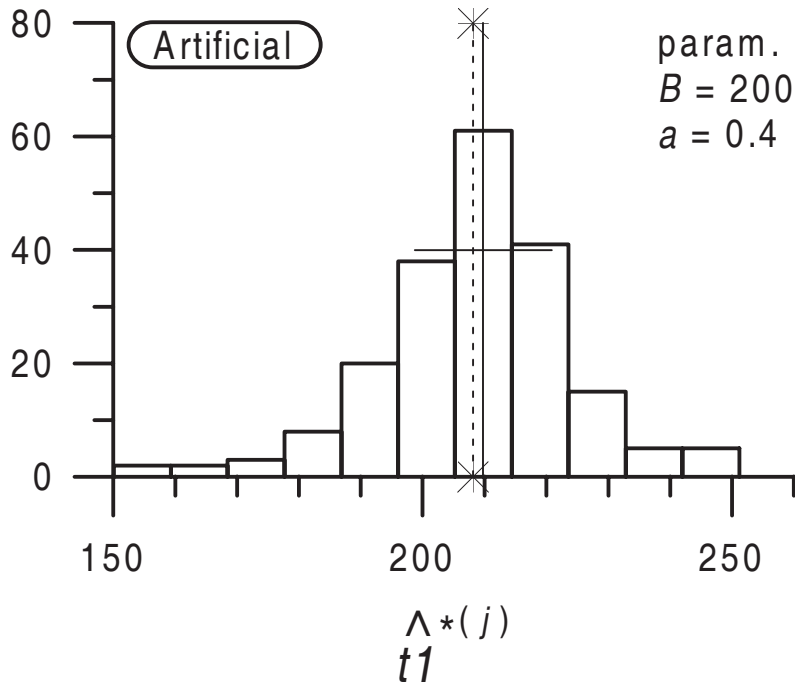


Figure 10: Histogram of $\hat{t}^{*(j)}$, parametric bootstrap, artificial time series. The vertical, dashed line (with stars) marks \hat{t} . The crossed, solid lines mark $med[\hat{t}^{*(j)}] \pm mad'[\hat{t}^{*(j)}]$.

where $e(1)^*$ is picked at random from $\{e(k)\}$. The other $e(i)^*$ are derived as follows. Let $e(i)^* = e(l)$, say. Then:

$$e(i+1)^* = \begin{cases} \text{with probability } p : \\ e(l+1) \text{ or, if } l = n, e(1), \\ \text{with probability } 1 - p : \\ \text{picked at random from } \{e(k)\}. \end{cases}$$

The $e(i)^*$ consist of blocks of variable length which preserve a sub-chain of the $e(i)$. That recognizes eventual autocorrelation of the $\epsilon(i)$. The probability p has to be prescribed. RAMPFIT suggests a value for which the average block length becomes equal to the estimated autocorrelation decay period $\hat{\tau}$.

4.3 Wild Bootstrap

The wild bootstrap of Härdle and Marron (1991), adapted to hetero-

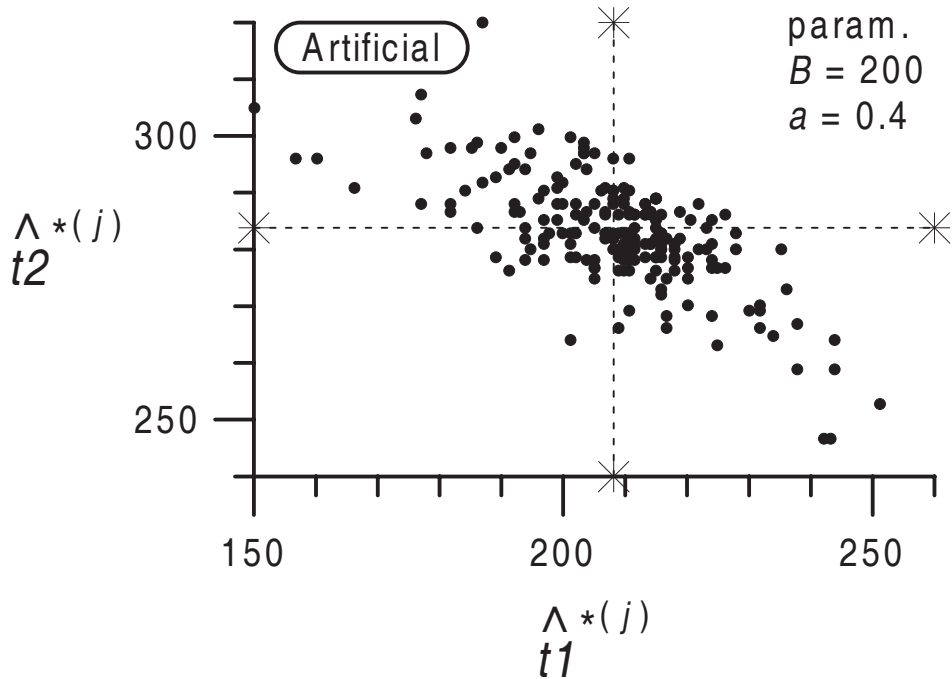


Figure 11: Scatterplot between $\hat{t}_1^{*(j)}$ and $\hat{t}_2^{*(j)}$, parametric bootstrap, artificial time series. The crossed, dashed lines (with stars) mark \hat{t}_1 and \hat{t}_2 , respectively.

scedasticity, resamples as

$$x(i)^* = \hat{x}_{\text{fit}}(i) + \hat{\sigma}(i) e(i) w(i), \quad i = 1, \dots, n,$$

where $w(i)$ is the two-point distribution,

$$w(i) = \begin{cases} \text{with probability } r : w_1, \\ \text{with probability } 1 - r : w_2, \end{cases}$$

with $r = (\sqrt{5} + 1)/(2\sqrt{5})$, $w_1 = (1 - \sqrt{5})/2$, and $w_2 = (1 + \sqrt{5})/2$. The wild bootstrap attempts “to reconstruct the distribution of each residual through the use of one single observation” (Härdle and Marron, 1991). This resampling scheme, in contrast to the two others, preserves no dependency between the residuals.

The question of which of the resampling schemes may be most adequate for ramp function regression has to be pursued further, for example, by means of Monte Carlo simulations. At present, I conjecture that eventual deviations

Table 2. Analyzed time series

Name	Description	Time interval	n	V
Artificial	$t1 = 200, x1 = 2.0$ $t2 = 300, x2 = 4.0$ $\sigma(t) = 1.0 (t \leq 250)$ $0.5 (t > 250)$ $a = 0.40$	[0; 499]	500	0.208
ODP 925	$\delta^{18}\text{O}$ [per mil vs. PDB], benthic foraminifera	[500 ka; 1400 ka]	429	0.786
87-Sr/86-Sr	87-Sr/86-Sr, various cores, planktonic foraminifera	[2.0 Ma; 8.8 Ma]	65	1.941
ODP 929	$\delta^{18}\text{O}$ [per mil vs. PDB], benthic foraminifera	[23.4 Ma; 23.8 Ma] [23.7 Ma; 24.6 Ma]	70 150	0.165 0.158

Notes: Time units refer to years before present. V is the coefficient of variation of the time spacing, that is, the standard deviation divided by the average. The ODP 929 time series has been splitted in two parts.

from Gaussianity or AR(1) dependency are best preserved by the nonparametric stationary bootstrap.

An eventual important addition to RAMPFIT would be the incorporation of a scheme which also resamples the time $t(i)$. An uncertain timescale is most common for paleoclimatic time series. It may, however, be necessary to develop a statistical model for the $t(i)$.

5 Examples

After having tested ramp function regression, residual analysis, and bootstrap resampling for an artificial time series, we investigate three measured marine, paleoclimatic time series. Table 2 describes the data sets and Table 3 (Appendix A) presents the results. The fits are plotted here whereas residual analysis and bootstrap plots are given in Appendix A.

5.1 ODP 925

The ODP 925 $\delta^{18}\text{O}$ record (Bickert, Curry and Wefer, 1997) documents

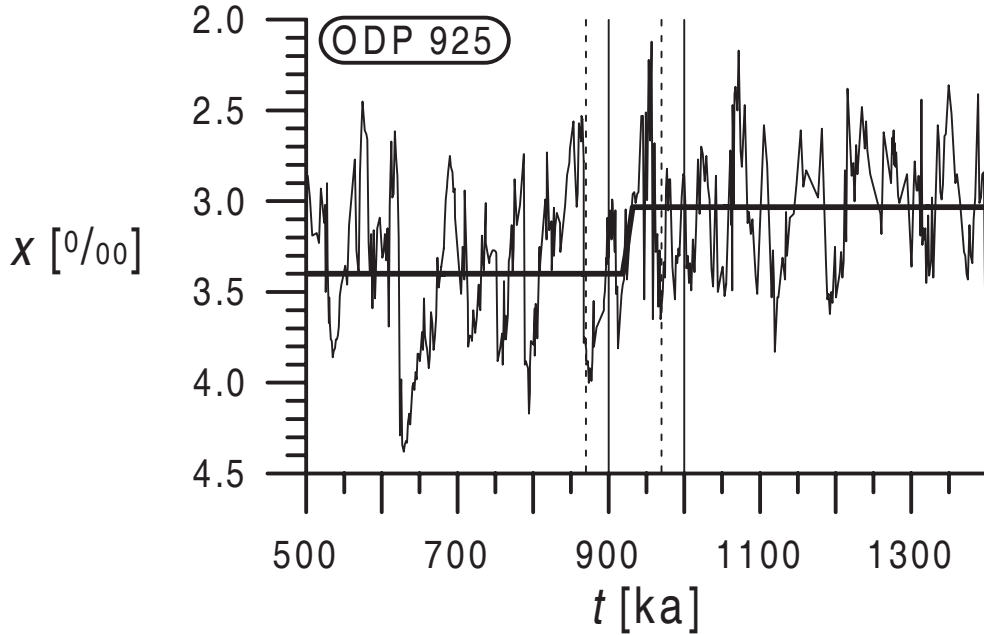


Figure 12: Ramp function fit (heavy line) to the ODP 925 time series (Tables 2 and 3). Grid boundaries for the t_1 -search (dashed vertical lines) and the t_2 -search (solid).

growth and decay of Northern Hemisphere ice mass. High $\delta^{18}\text{O}$ values mean large ice mass. It contains the Mid-Pleistocene Transition (MPT), a relatively sudden increase in mean ice mass at around 920 ka before present (BP) which led to Late Pleistocene ice ages (see Mudelsee and Stattegger (1997) and also Table 1 therein for further references). For fit region, we choose the same time interval, [500 ka; 1400 ka], as Mudelsee and Stattegger (1997) for other records.

The ramp function fit (Fig. 12, Table 3) confirms timing and abruptness of the MPT. The estimated $\delta^{18}\text{O}$ step agrees with results from other records. The location of the ODP 925 drill hole (tropical W-Atlantic) adds further evidence that the MPT was a global climate phenomenon. Residual analysis attests that made assumptions are valid and the ramp form is suited. Dependency between the $e(i)$ is considerably strong ($\hat{a} = 0.805$) which may be due to harmonic components (Milankovitch frequencies of Earth orbital parameter variations). That deviation from AR(1) dependency should make

the nonparametric bootstrap (we take $p = 0.7$) the most reliable scheme. However, all three resampling methods reveal nearly zero bias of estimated parameters and no correlation between them (exception: $\hat{t}1^{*(j)} < \hat{t}2^{*(j)}$). Estimated parameter uncertainties (Table 3) roughly agree with those of previous quantifications of the MPT on other records (Mudelsee and Schulz, 1997; Mudelsee and Stattegger, 1997). The estimated uncertainties of $\hat{t}1$ and $\hat{t}2$ are about 7–10 times as large as the average spacing, justifying the coarse search grid.

5.2 87-Sr/86-Sr

The 87-Sr/86-Sr record (Hodell and others, 1989), constructed from various marine sedimentary cores, documents the geochemical cycling of strontium in the late Neogene ocean which is related to processes such as chemical weathering of the continents, hydrothermal circulation at mid-ocean ridges, and carbonate dissolution on the seafloor. Hodell and others fitted a ramp function per eye ($\hat{t}1 = 4.5$ Ma, $\hat{x}1 = 0.709025$, $\hat{t}2 = 5.5$ Ma, $\hat{x}2 = 0.708925$) and concluded that the 87-Sr/86-Sr record additionally offers considerable potential as stratigraphic tool for the transition interval. As $\sigma(t)$ we overtake the reported measurement errors.

The ramp function regression carried out with RAMPFIT (Fig. 13, Table 3) confirms the findings of Hodell and others. Residual analysis attests that made assumptions are valid and the ramp form is suited. There is no dependency between the $e(i)$, and all three resampling schemes reveal the following similar results: negligible biases for the estimated parameters and minor correlation between them. The estimated uncertainties of $\hat{t}1$ and $\hat{t}2$ (Table 3) are about as large as the average spacing, which seems to be still acceptable.

5.3 ODP 929

The ODP 929 $\delta^{18}\text{O}$ time series (Zachos, Flower and Paul, 1997) from the tropical W-Atlantic is a high-resolution record of Global Change over the Oligocene/Miocene boundary. Climate was moderately warm but punctuated by episodes of global cooling. The first and largest such episode is Mi-1, at around 23.8 Ma BP. The ODP 929 record mainly reflects changes in bottom water temperature (signal proportion $> 2/3$ (Zachos, 1998, pers. comm.)), which enables us to quantify the Mi-1 event. High $\delta^{18}\text{O}$ values

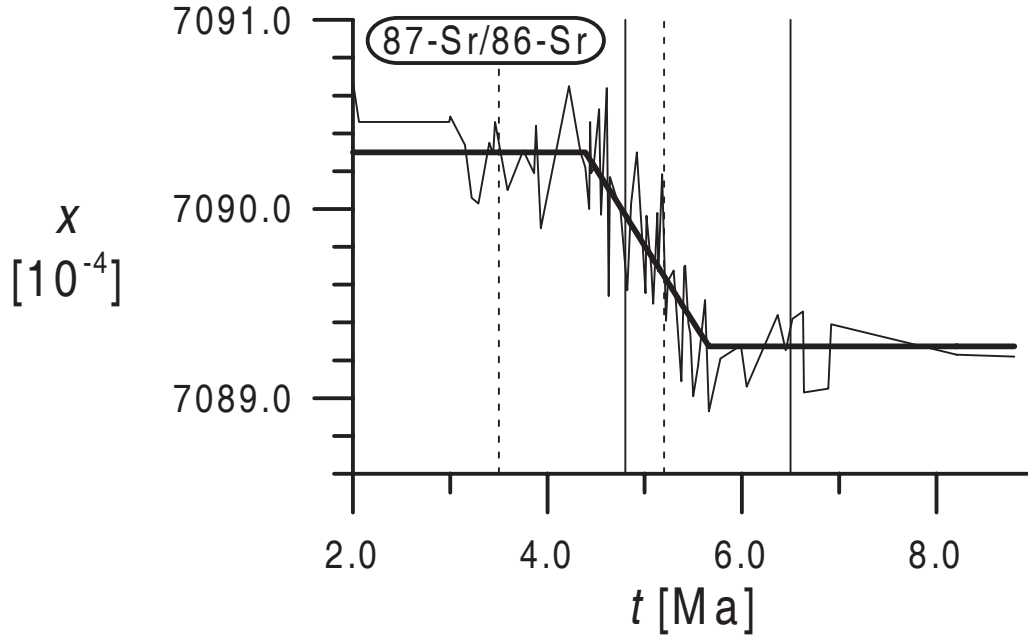


Figure 13: Ramp function fit (heavy line) to the $87\text{-Sr}/86\text{-Sr}$ time series (Tables 2 and 3). Grid boundaries for the t_1 -search (dashed vertical lines) and the t_2 -search (solid).

mean low temperature. We use the same time interval, [23.4 Ma; 24.6 Ma], as Zachos, Flower and Paul (1997: Fig. 1b therein) for analyzing Mi-1.

A *single* ramp function would not provide an adequate model for this data. This can be seen directly from Figure 14 and would also be revealed by residual analysis. Instead we fit *two* ramps, the older transition corresponding to initial, strong cooling, the younger to the following, minor warming (Fig. 14, Table 3). Residual analysis for the old part (Appendix A) generally confirms the ramp form. However, the Gaussian assumption is eventually violated. Likewise, the regression model cannot be rejected for the young part. But, a good alternative might be a harmonic model for the mean which would describe the Mi-1 event as part of a 400 ka Milankovitch cycle, see also Zachos, Flower and Paul (1997). Bootstrap resampling does not indicate estimation bias, except perhaps of minor degree for \hat{x}_1 and \hat{x}_2 in the old part. For both parts the estimated uncertainties of \hat{t}_1 and \hat{t}_2 are about 3 times as large as the average spacing.

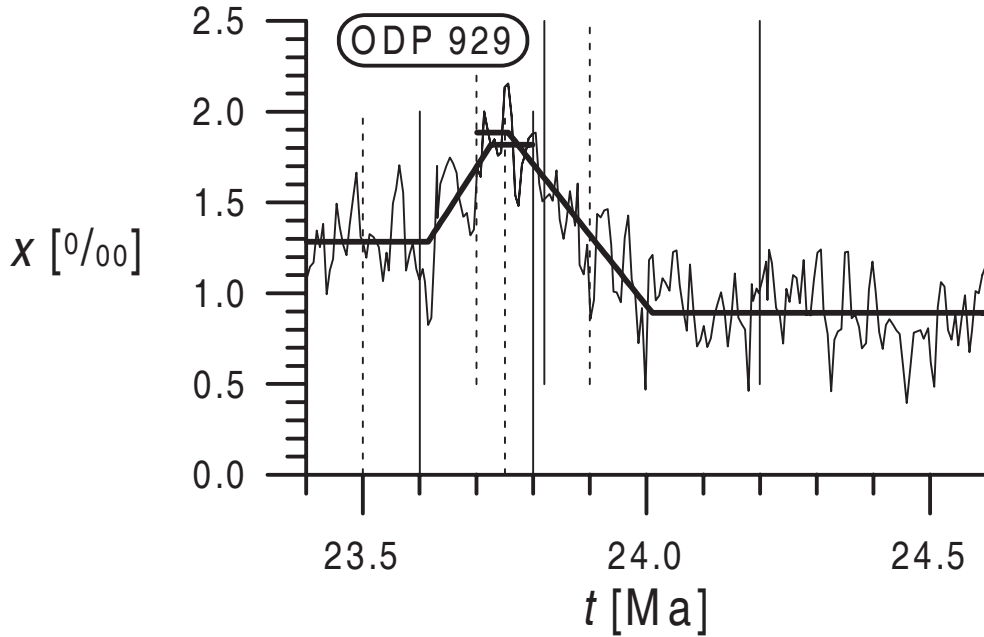


Figure 14: Two-fold ramp function fit (heavy line) to the ODP 929 time series (Tables 2 and 3). Grid boundaries for the t_1 -searches (dashed vertical lines, those for the young part border on t -axis) and the t_2 -searches (solid).

6 Conclusions and Summary

I have presented a method, accompanied by a Fortran 77 computer program (RAMPFIT), for fitting a ramp function (Fig. 1) to measured paleoclimatic time series.

- A brute-force search over a grid (Fig. 2) estimates t_1 and t_2 , whereas a weighted LS estimation provides \hat{x}_1 and \hat{x}_2 . An intelligent search path reduces costs. RAMPFIT's interactive graphics further facilitate keeping the grid boundaries narrow. The example time series (1 artificial, 3 measured; $n \leq 500$) show the practicability of this approach in terms of computing costs and precision of the solution: Estimated parameter uncertainties were greater than the grids were coarse.
- $\sigma(t)$ is fitted per eye, also as a ramp function. For the example time series, this seems to be sufficient for providing approximate weights,

including when dependency is present.

- Interesting extensions/alternative approaches would be: (1) generalized LS, (2) numerical minimization, (3) a continuously differentiable approximation of the ramp function, and (4) LS fitting of $\sigma(t)$.

RAMPFIT includes an analysis of weighted ordinary residuals for testing whether made assumptions (namely Gaussian distribution, appropriate weighting, model suitability, and only weak AR(1) dependency) are valid. In case of the measured climate time series (Table 2), we found that:

- The deviations from Gaussianity are of only minor degree.
- The ramp function proved to be a suitable model. In one case (ODP 929) its form was too simple and had to be changed for a two-fold ramp.
- Serial dependency between the $e(i)$ might be strong in the presence of harmonic components (ODP 925).

RAMPFIT supplies three bootstrap resampling schemes (parametric, non-parametric stationary, and wild) for determining the accuracy of estimated parameters.

- For the measured time series, the three schemes produced similar results. It remains to be investigated further which resampling scheme is most suitable. Using one scheme alone should be sufficient. The non-parametric stationary bootstrap might be favorable because it might best tolerate deviations from Gaussianity or AR(1) dependency.
- It is recommended that 200 bootstrap replications (Efron and Tibshirani, 1993) be used. For typical paleoclimatic time series, this demands computing power at the Pentium/workstation level.

To summarize, ramp function regression seems to be a valuable tool for quantifying climate transitions on measured time series. Such a functional form is not uncommon in the geologic past. Modern computer power allows practical determination of parameter uncertainties.

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A Additional Material

Table 3. Results

Artificial										
Fits		$\hat{t}1$	$\hat{x}1$	$\hat{t}2$	$\hat{x}2$	$SSQWN$	\hat{a}	$\hat{\tau}$		
	$\hat{x}_{\text{fit}}(t)$	208.149	1.814	283.834	3.975	1.084	0.409	1.119		
	$\hat{\sigma}(t)$	250.000	0.900	250.000	0.500					
Parametric bootstrap, $B = 200$, $a = 0.400$										
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>	
<i>t1</i>	208.149	208.420	15.172	209.786	11.064	150.067	251.193	1	0	
<i>x1</i>	1.814	1.810	0.103	1.813	0.100	1.538	2.102			
<i>t2</i>	283.834	283.146	9.904	282.878	7.072	246.852	319.822	0	0	
<i>x2</i>	3.975	3.980	0.058	3.977	0.059	3.806	4.133			
Nonparametric bootstrap, $B = 200$, $p = 0.500$										
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>	
<i>t1</i>	208.149	207.643	12.273	209.163	10.097	166.175	241.172	0	0	
<i>x1</i>	1.814	1.797	0.079	1.797	0.079	1.572	2.003			
<i>t2</i>	283.834	283.247	7.449	283.834	6.934	262.947	312.973	0	0	
<i>x2</i>	3.975	3.973	0.042	3.977	0.041	3.864	4.074			
Wild bootstrap, $B = 200$										
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>	
<i>t1</i>	208.149	209.316	8.374	209.163	8.787	185.249	228.956	0	0	
<i>x1</i>	1.814	1.815	0.063	1.819	0.061	1.656	1.994			
<i>t2</i>	283.834	282.855	5.239	282.878	4.680	270.822	296.122	0	0	
<i>x2</i>	3.975	3.977	0.035	3.977	0.038	3.869	4.054			

Notes: We have neglected to introduce new parameter names in case of the ramp function fit to $\sigma(t)$. *nl* and *nr* count the bootstrap replications that yield a solution on the left (lower) and the right boundary, respectively, of the search grid (Fig. 2).

Table 3. (Continued)

ODP 925									
Fits	$t\hat{1}$	$x\hat{1}$	$t\hat{2}$	$x\hat{2}$	$SSQWN$	\hat{a}	$\hat{\tau}$		
$x_{\text{fit}}(t)$	918.730	3.399	931.780	3.032	1.018	0.805	4.602		
$\hat{\sigma}(t)$	650.000	0.450	1000.000	0.350					
Parametric bootstrap, $B = 200$, $a = 0.800$									
	fit	ave	std	med	mad'	min	max	nl	nr
$t1$	918.730	917.293	25.935	920.510	22.372	870.229	968.710	16	6
$x1$	3.399	3.414	0.069	3.415	0.072	3.199	3.616		
$t2$	931.780	937.501	26.335	931.780	19.585	900.183	1000.000	17	7
$x2$	3.032	3.020	0.060	3.019	0.048	2.827	3.196		
Nonparametric bootstrap, $B = 200$, $p = 0.700$									
	fit	ave	std	med	mad'	min	max	nl	nr
$t1$	918.730	916.716	19.128	918.730	13.180	870.229	968.710	11	3
$x1$	3.399	3.408	0.035	3.409	0.032	3.310	3.511		
$t2$	931.780	936.559	20.863	931.780	14.070	900.183	1000.000	1	2
$x2$	3.032	3.027	0.033	3.028	0.033	2.927	3.120		
Wild bootstrap, $B = 200$									
	fit	ave	std	med	mad'	min	max	nl	nr
$t1$	918.730	912.836	17.066	918.730	8.036	870.229	968.710	3	1
$x1$	3.399	3.403	0.032	3.404	0.036	3.321	3.509		
$t2$	931.780	944.059	21.605	933.690	14.159	901.847	1000.000	0	5
$x2$	3.032	3.025	0.026	3.024	0.028	2.957	3.088		

Note: For the measured time series, the units of $\hat{\tau}$ are equal to those of $t(i)$.

Table 3. (Continued)

87-Sr/86-Sr									
Fits	$t\hat{1}$	$x\hat{1}$	$t\hat{2}$	$x\hat{2}$	$SSQWN$	\hat{a}	$\hat{\tau}$		
$x_{\text{fit}}(t)$	4.390	7090.300	5.660	7089.274	1.716	0.000			
$\hat{\sigma}(t)$	(measurement error)								
Parametric bootstrap, $B = 200$, $a = 0.000$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	4.390	4.399	.123	4.390	.074	3.930	4.920	0	0
<i>x1</i>	7090.300	7090.295	.040	7090.299	.039	7090.184	7090.432		
<i>t2</i>	5.660	5.665	.119	5.660	.163	5.300	6.050	0	0
<i>x2</i>	7089.274	7089.270	.048	7089.270	.050	7089.146	7089.385		
Nonparametric bootstrap, $B = 200$, $p = 1.000$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	4.390	4.397	.163	4.390	.119	3.930	4.820	0	0
<i>x1</i>	7090.300	7090.301	.058	7090.297	.062	7090.165	7090.461		
<i>t2</i>	5.660	5.662	.164	5.660	.178	5.220	6.370	0	0
<i>x2</i>	7089.274	7089.268	.071	7089.261	.061	7089.081	7089.468		
Wild bootstrap, $B = 200$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	4.390	4.413	.165	4.430	.133	3.590	4.920	1	0
<i>x1</i>	7090.300	7090.297	.057	7090.301	.052	7090.132	7090.456		
<i>t2</i>	5.660	5.649	.155	5.660	.170	5.180	6.050	0	0
<i>x2</i>	7089.274	7089.277	.043	7089.275	.038	7089.155	7089.408		

Note: x -values have been multiplied by 10^4 .

Table 3. (Continued)

ODP 929, young part, [23.4 Ma; 23.8 Ma]									
Fits	$\hat{t}1$	$\hat{x}1$	$\hat{t}2$	$\hat{x}2$	<i>SSQWN</i>	\hat{a}	$\hat{\tau}$		
$\hat{x}_{\text{fit}}(t)$	23.615	1.284	23.726	1.819	1.183	0.000			
$\hat{\sigma}(t)$	23.500	0.160	23.600	0.200					
Parametric bootstrap, $B = 200$, $a = 0.000$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	23.615	23.615	0.026	23.615	0.023	23.512	23.683	0	0
<i>x1</i>	1.284	1.282	0.028	1.286	0.025	1.190	1.364		
<i>t2</i>	23.726	23.728	0.032	23.726	0.027	23.659	23.798	0	10
<i>x2</i>	1.819	1.836	0.073	1.828	0.070	1.671	2.065		
Nonparametric bootstrap, $B = 200$, $p = 1.000$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	23.615	23.617	0.028	23.620	0.024	23.523	23.689	0	0
<i>x1</i>	1.284	1.285	0.031	1.283	0.029	1.197	1.377		
<i>t2</i>	23.726	23.725	0.034	23.719	0.027	23.659	23.798	0	9
<i>x2</i>	1.819	1.824	0.072	1.821	0.066	1.644	2.044		
Wild bootstrap, $B = 200$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	23.615	23.618	0.029	23.620	0.030	23.500	23.695	1	0
<i>x1</i>	1.284	1.285	0.032	1.283	0.035	1.208	1.366		
<i>t2</i>	23.726	23.720	0.026	23.716	0.031	23.659	23.798	0	1
<i>x2</i>	1.819	1.819	0.058	1.813	0.060	1.692	1.976		

Table 3. (Continued)

ODP 929, old part, [23.7 Ma; 24.6 Ma]							
Fits	$\hat{t}1$	$\hat{x}1$	$\hat{t}2$	$\hat{x}2$	$SSQWN$	\hat{a}	$\hat{\tau}$
$\hat{x}_{\text{fit}}(t)$	23.756	1.884	24.010	0.892	1.058	0.000	
$\hat{\sigma}(t)$		0.190	(constant)				

Parametric bootstrap, $B = 200$, $a = 0.000$									
	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	23.756	23.751	0.023	23.750	0.027	23.701	23.816	3	0
<i>x1</i>	1.884	1.948	0.065	1.944	0.068	1.794	2.172		
<i>t2</i>	24.010	24.013	0.017	24.010	0.018	23.974	24.071	0	0
<i>x2</i>	0.892	0.877	0.019	0.877	0.019	0.818	0.921		

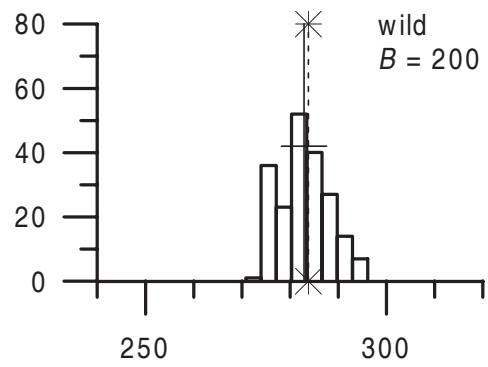
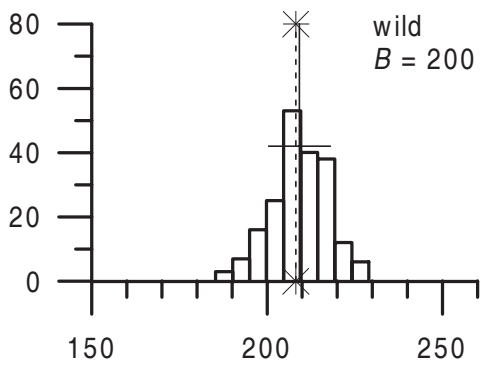
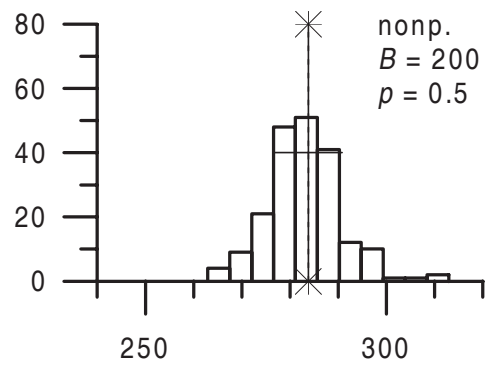
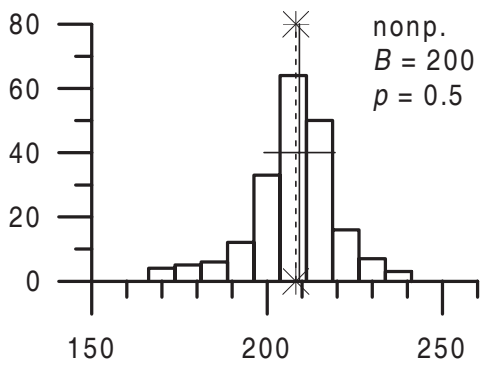
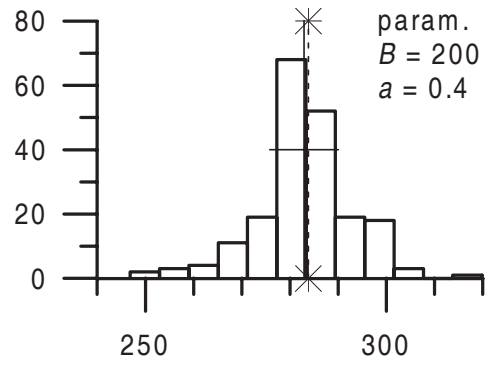
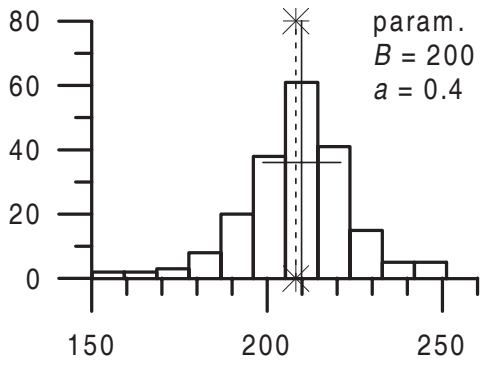
Nonparametric bootstrap, $B = 200$, $p = 1.000$

	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	23.756	23.748	0.026	23.750	0.027	23.701	23.804	7	0
<i>x1</i>	1.884	1.959	0.081	1.951	0.083	1.765	2.134		
<i>t2</i>	24.010	24.014	0.016	24.010	0.018	23.986	24.083	0	0
<i>x2</i>	0.892	0.880	0.021	0.880	0.022	0.830	0.931		

Wild bootstrap, $B = 200$

	<i>fit</i>	<i>ave</i>	<i>std</i>	<i>med</i>	<i>mad'</i>	<i>min</i>	<i>max</i>	<i>nl</i>	<i>nr</i>
<i>t1</i>	23.756	23.751	0.023	23.750	0.027	23.707	23.804	0	0
<i>x1</i>	1.884	1.946	0.061	1.949	0.056	1.752	2.104		
<i>t2</i>	24.010	24.020	0.020	24.013	0.022	23.986	24.077	0	0
<i>x2</i>	0.892	0.876	0.021	0.875	0.021	0.816	0.931		

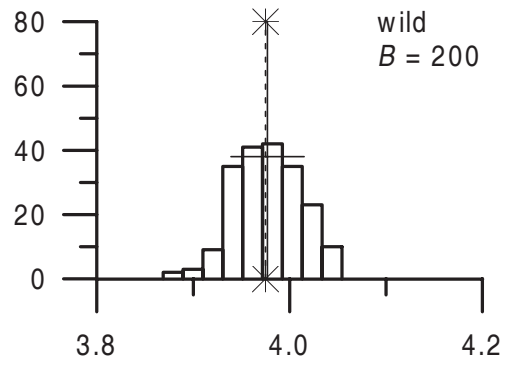
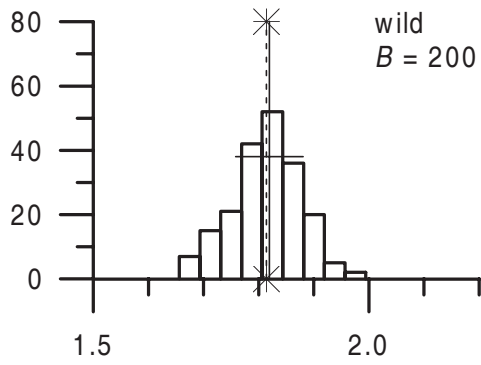
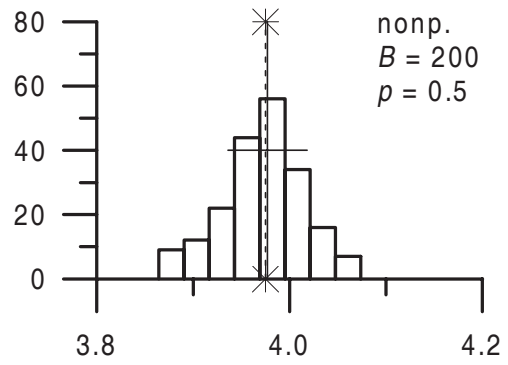
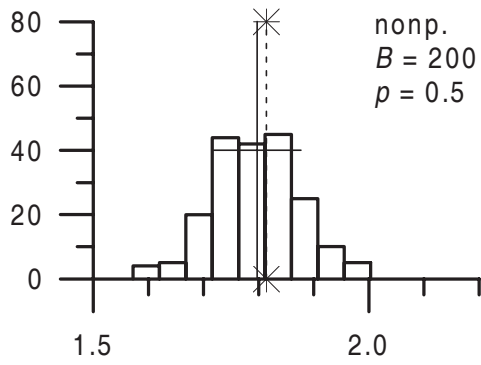
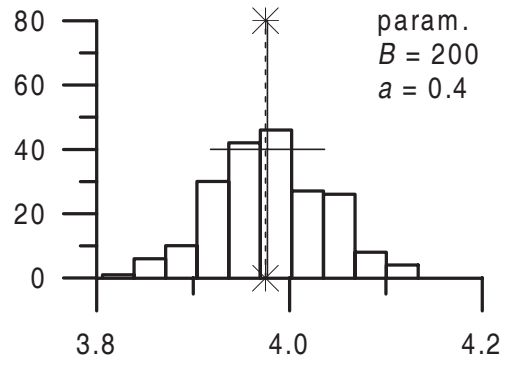
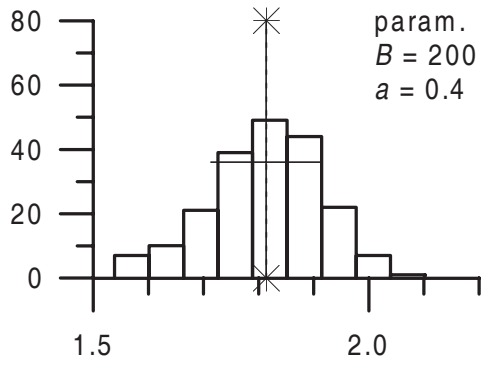
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$\Lambda^*(j)$
 $t1$

$\Lambda^*(j)$
 $t2$

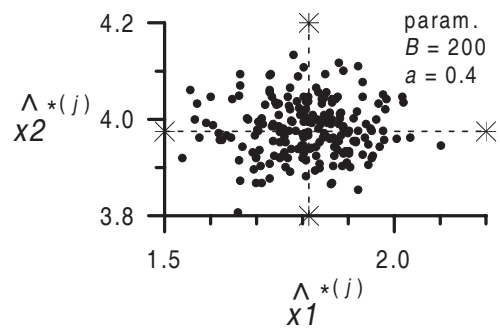
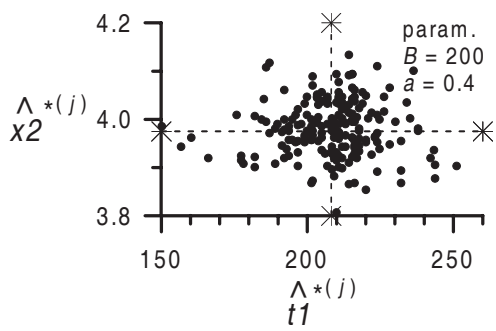
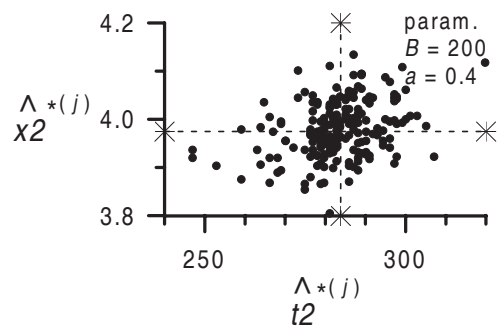
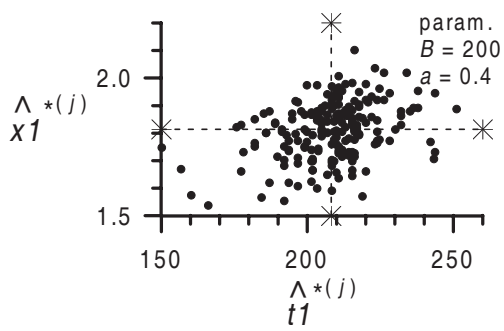
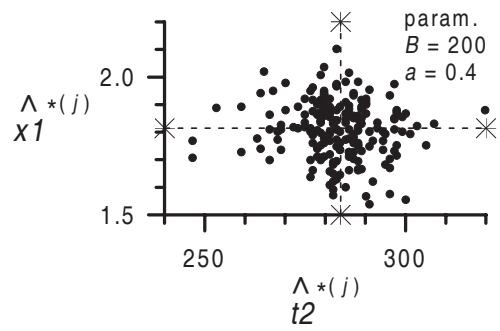
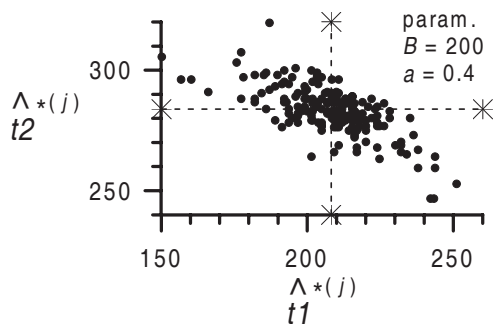
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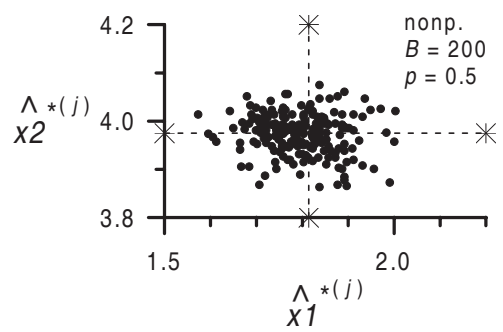
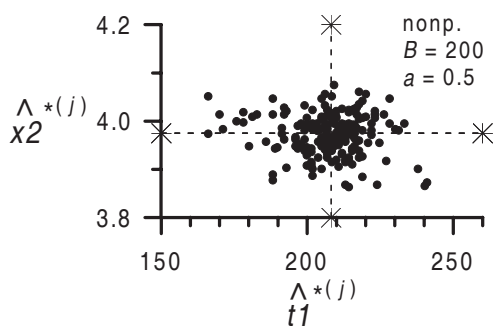
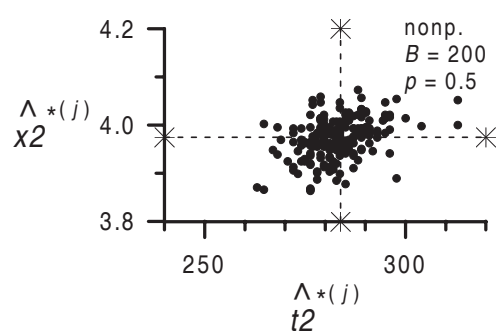
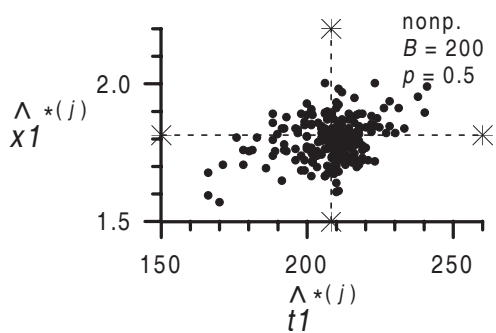
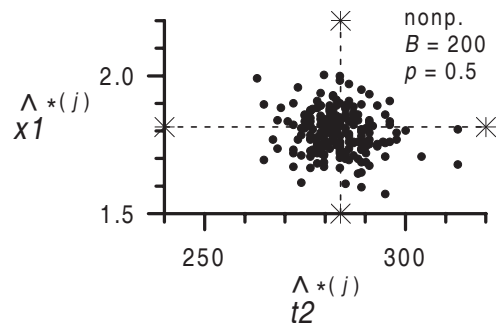
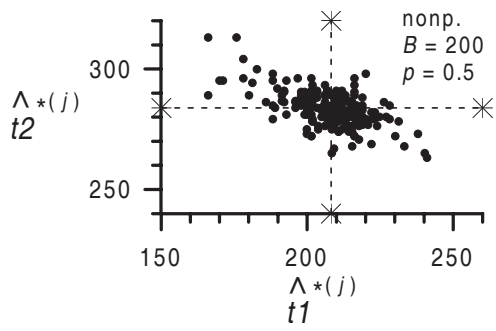
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 $x1$

$\Lambda^{*(j)}$
 $x2$

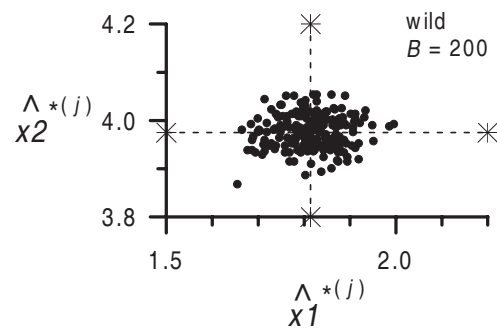
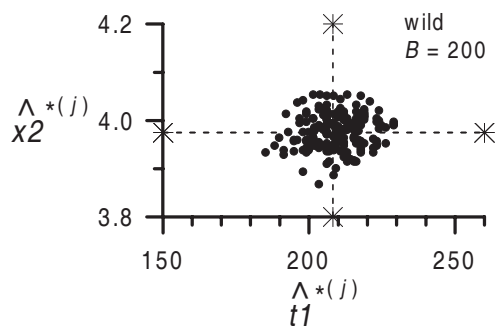
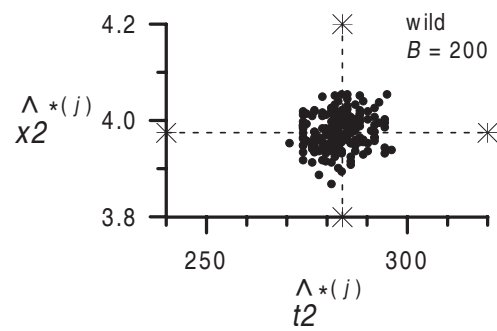
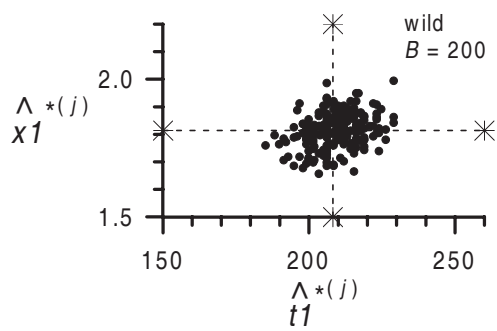
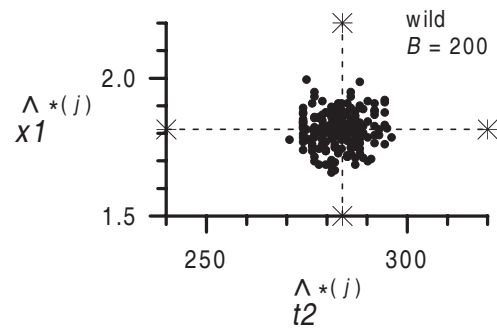
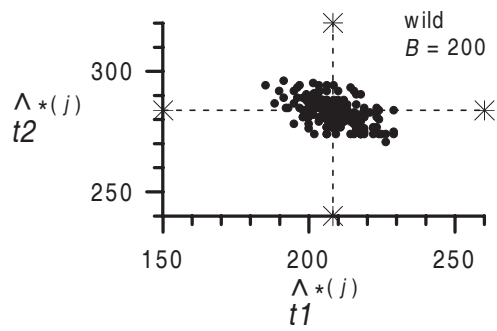
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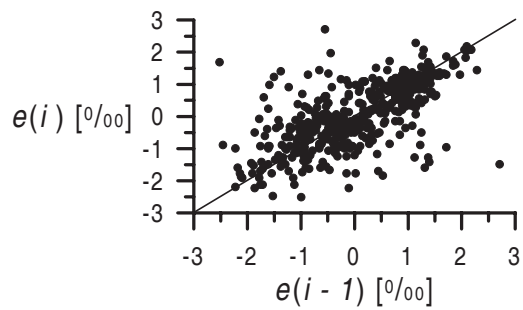
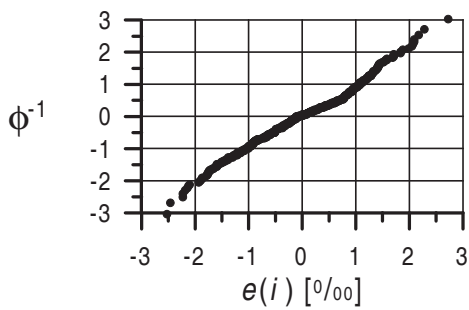
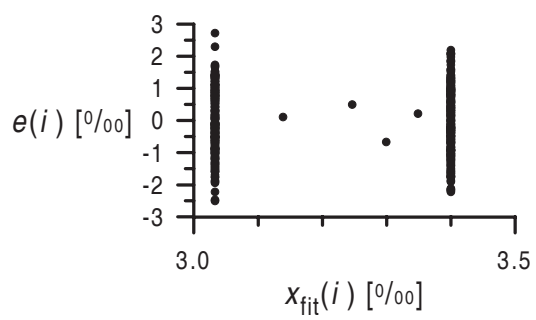
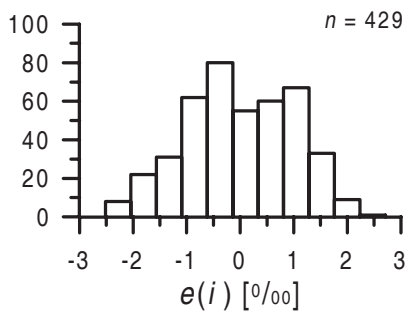
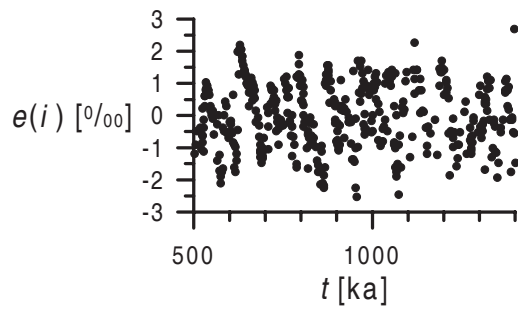
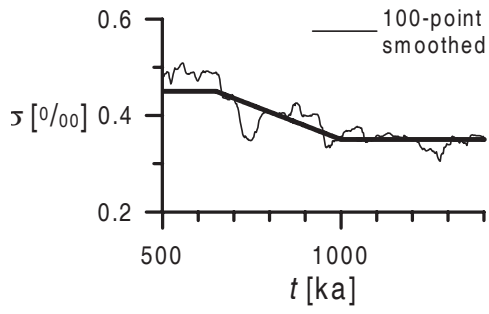
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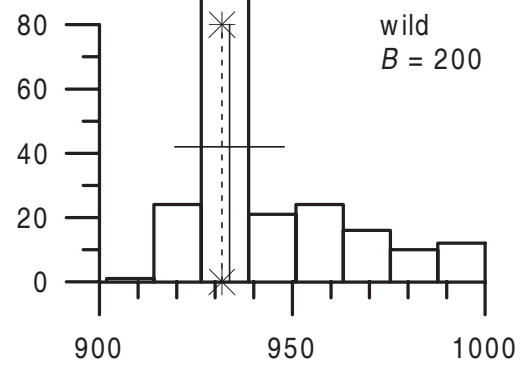
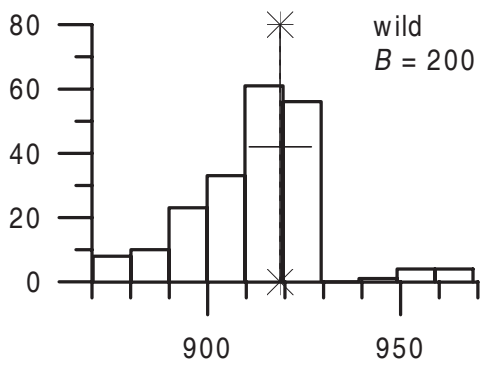
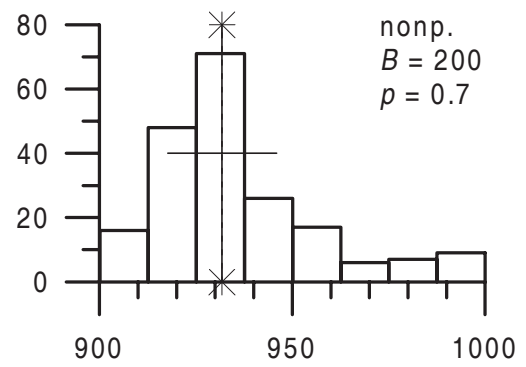
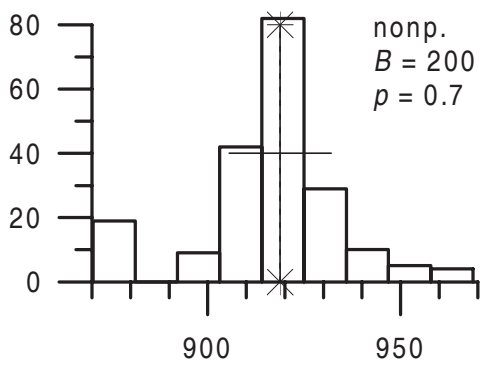
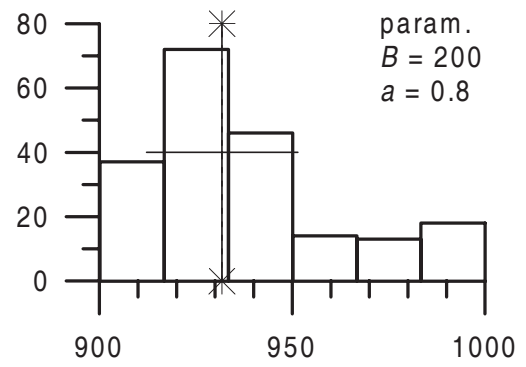
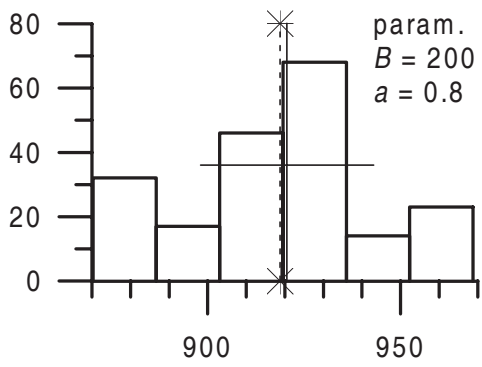
Artificial



ODP 925



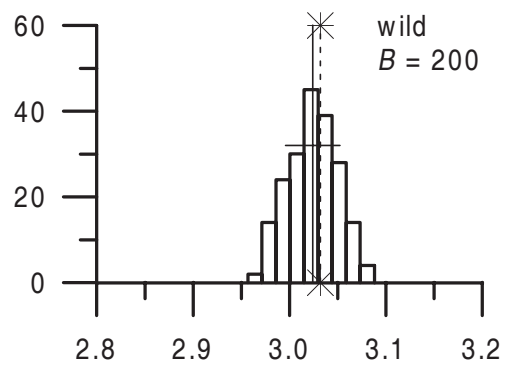
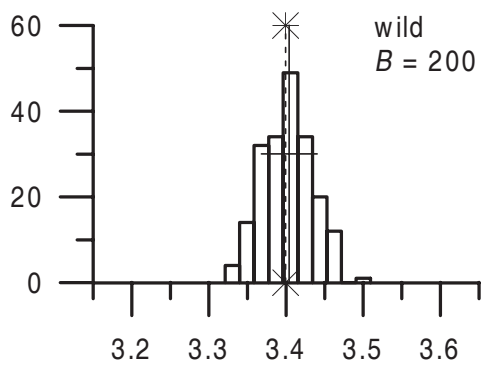
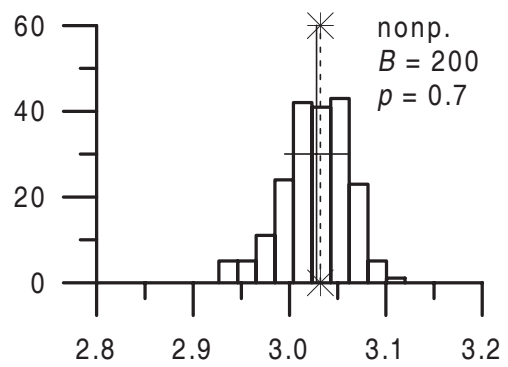
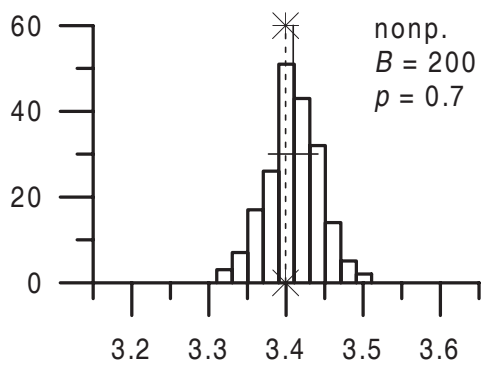
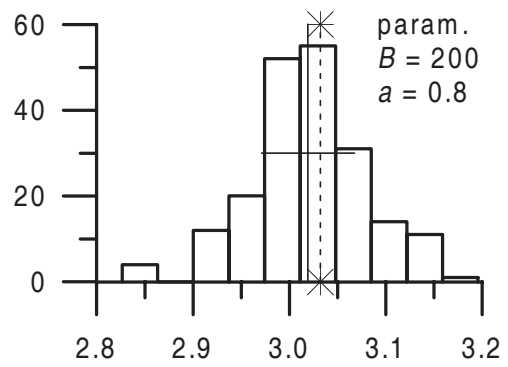
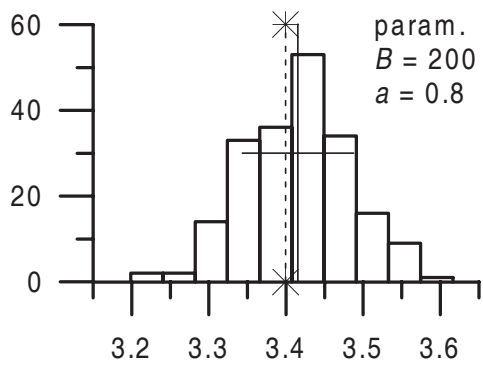
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 $t1$ [ka]

$\Lambda^{*(j)}$
 $t2$ [ka]

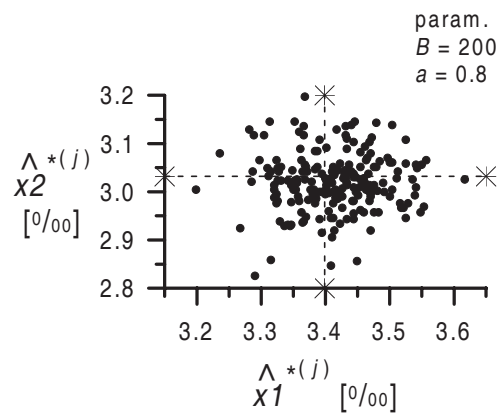
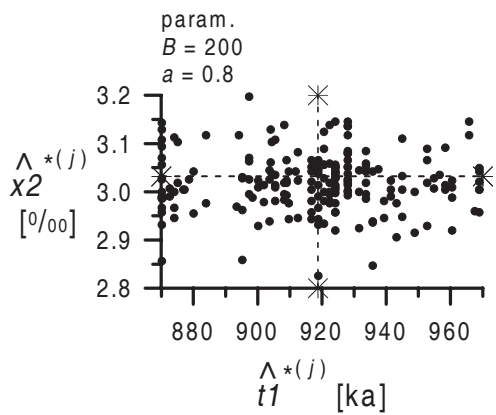
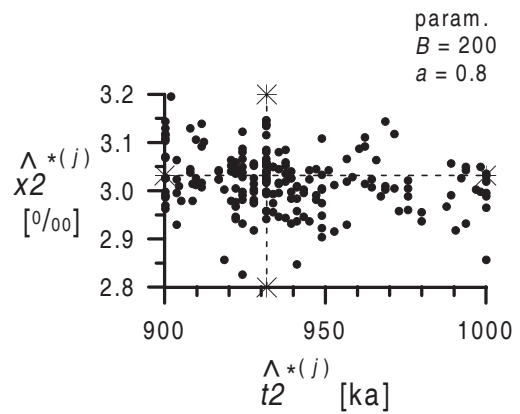
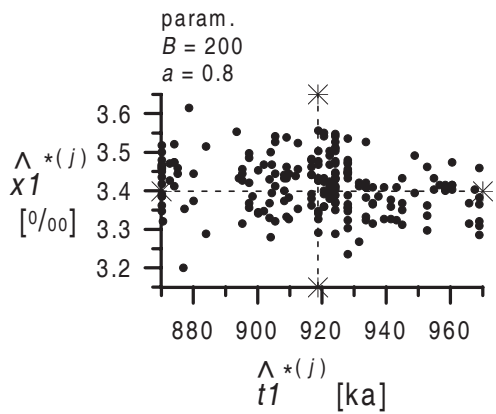
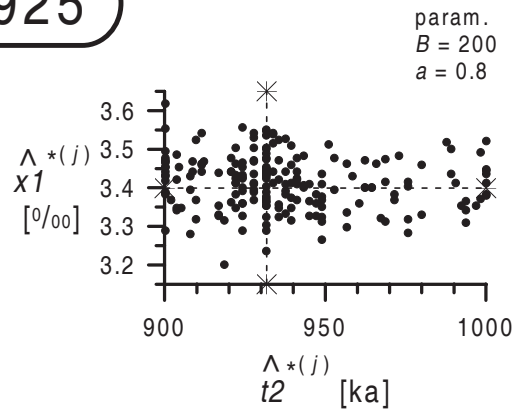
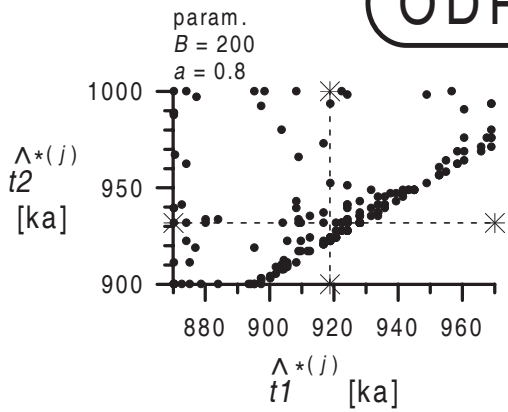
ODP 925



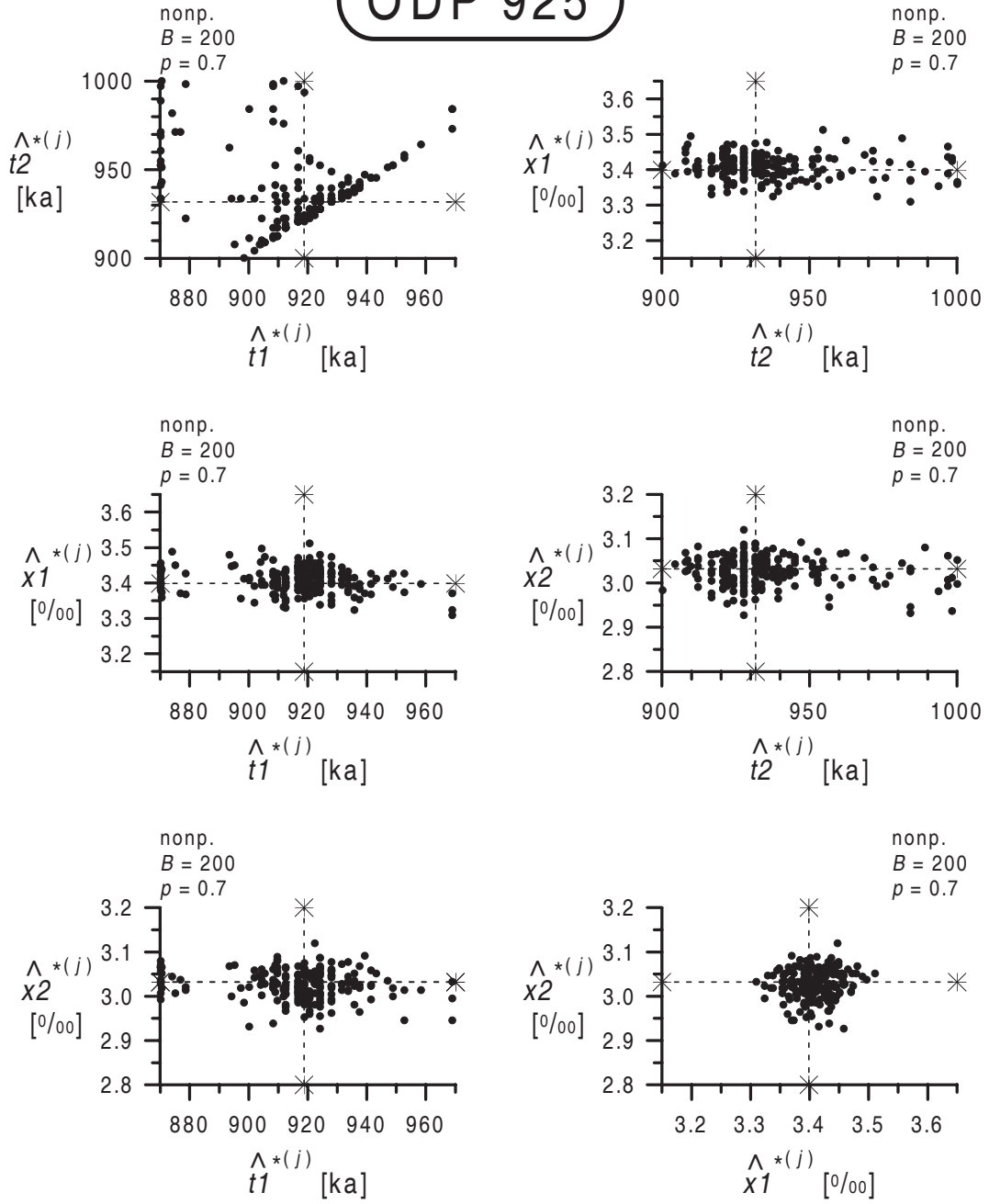
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$\Lambda^{*(j)}$
 $x2$ [0/00]

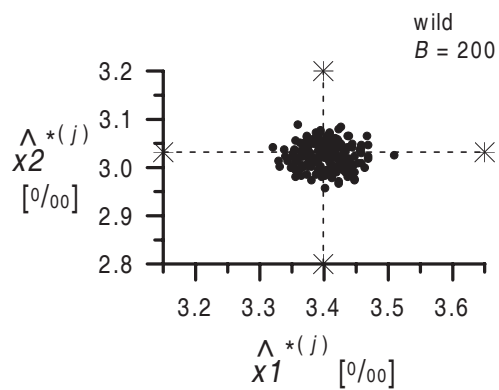
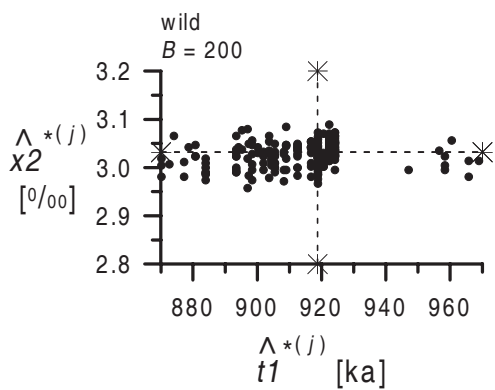
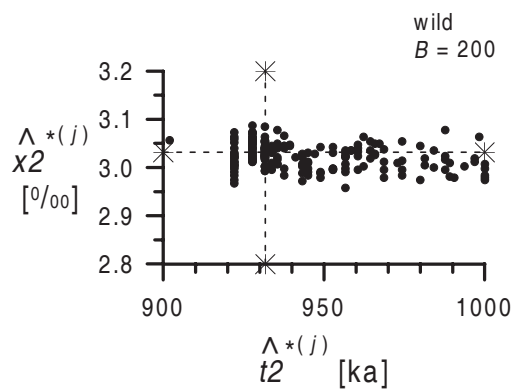
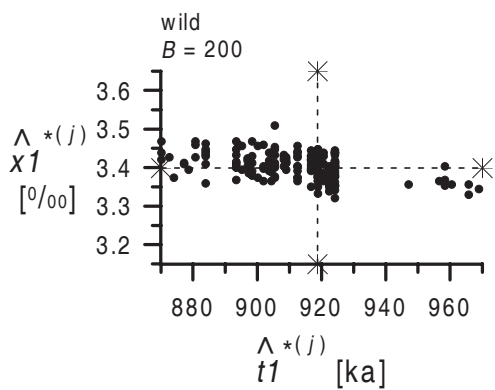
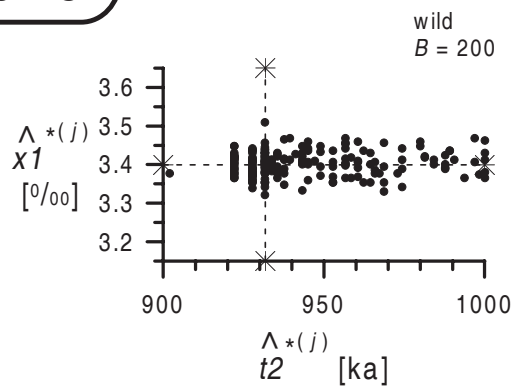
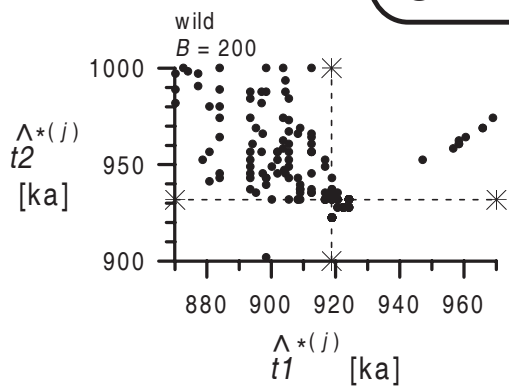
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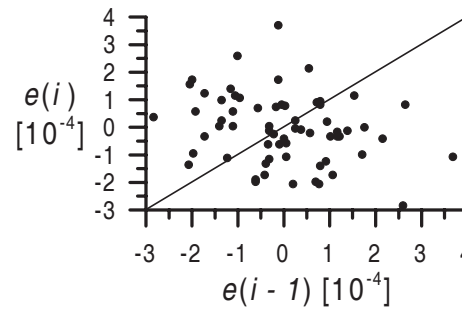
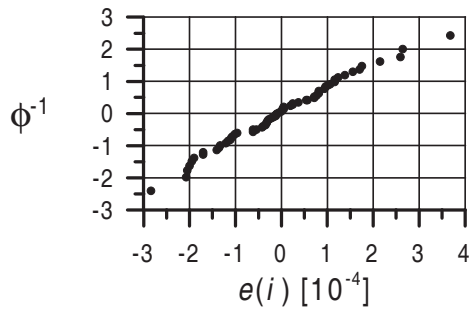
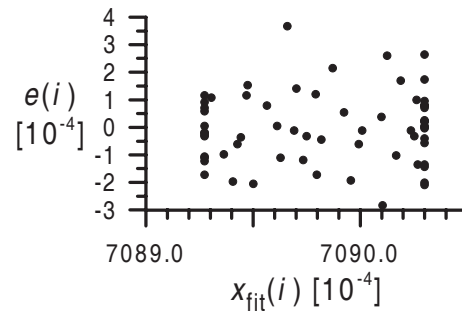
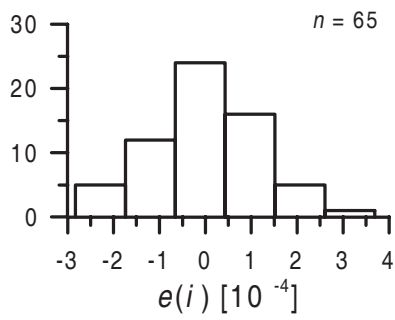
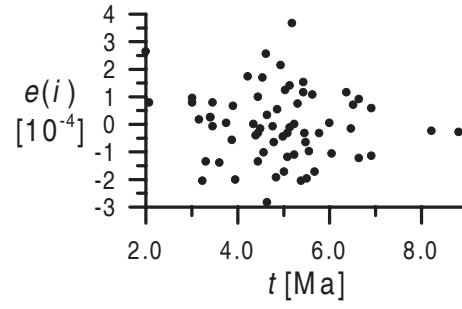
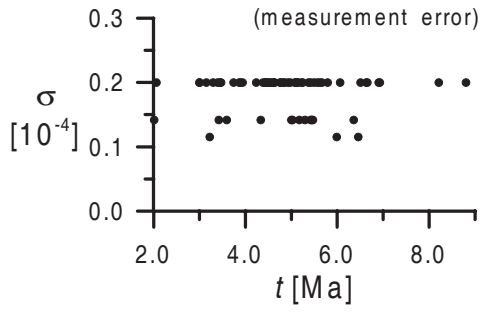
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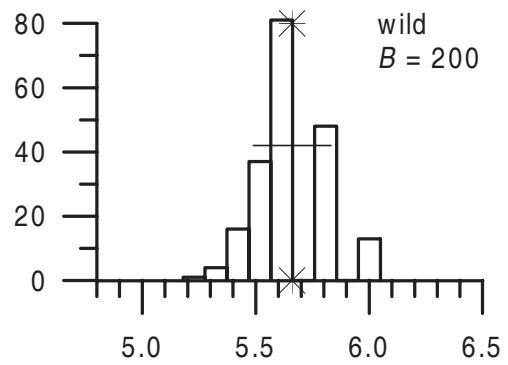
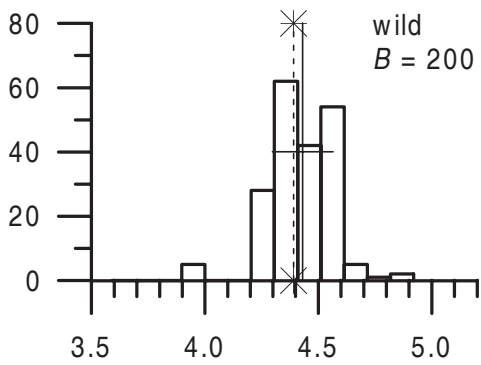
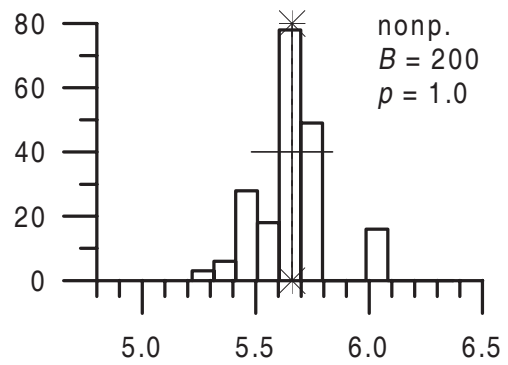
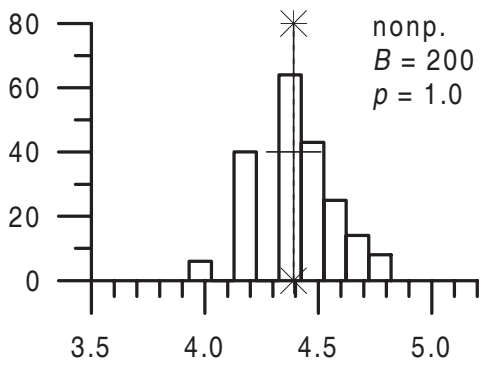
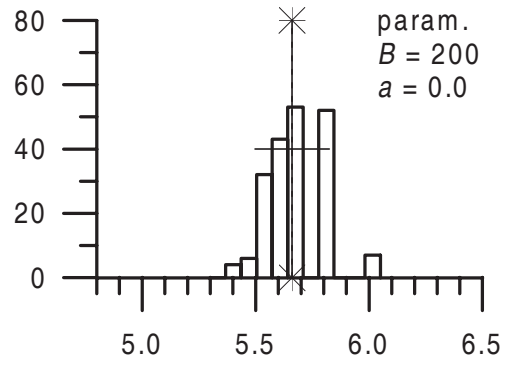
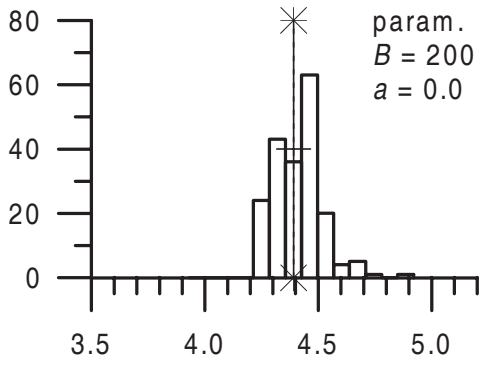
ODP 925



87-Sr/86-Sr



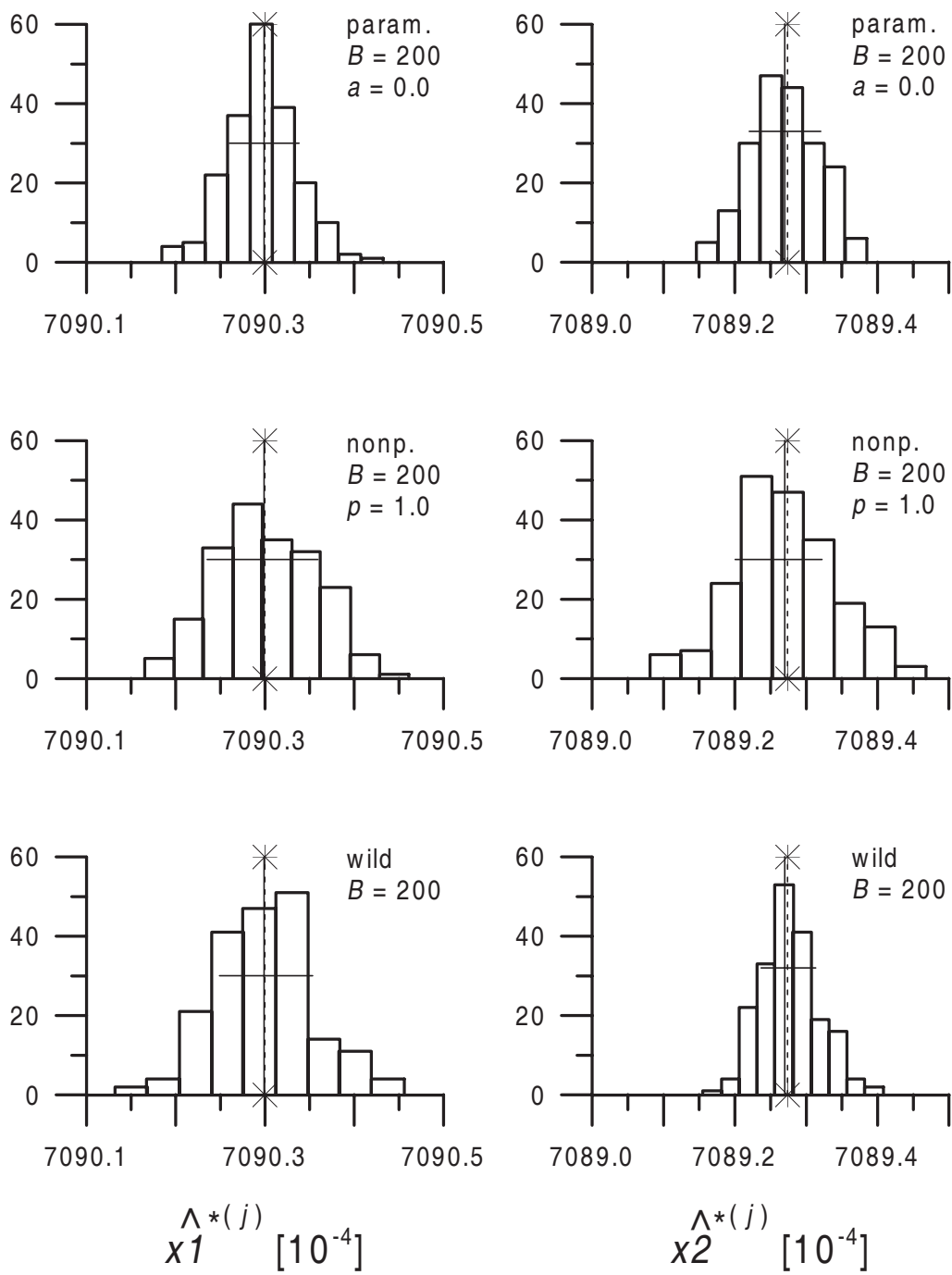
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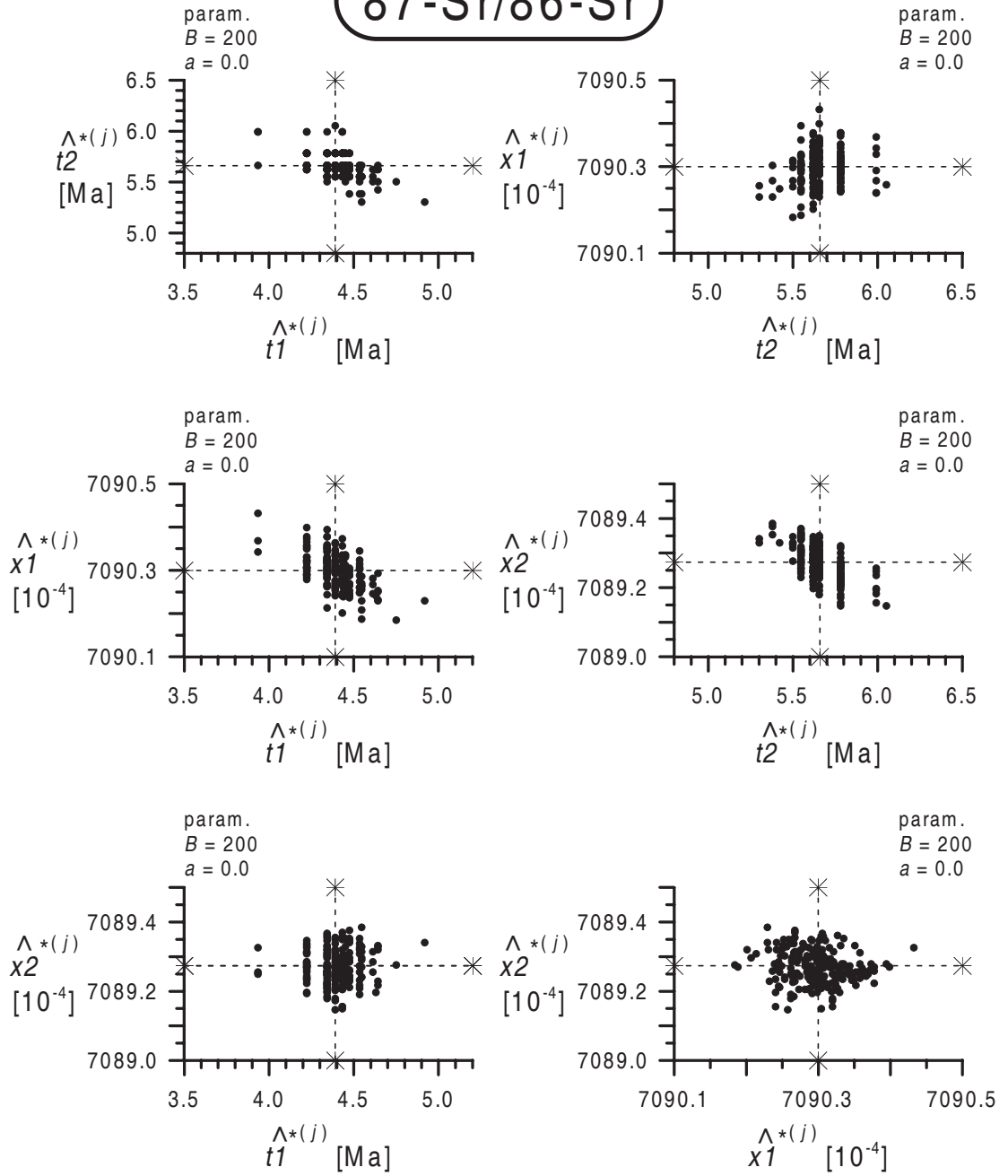
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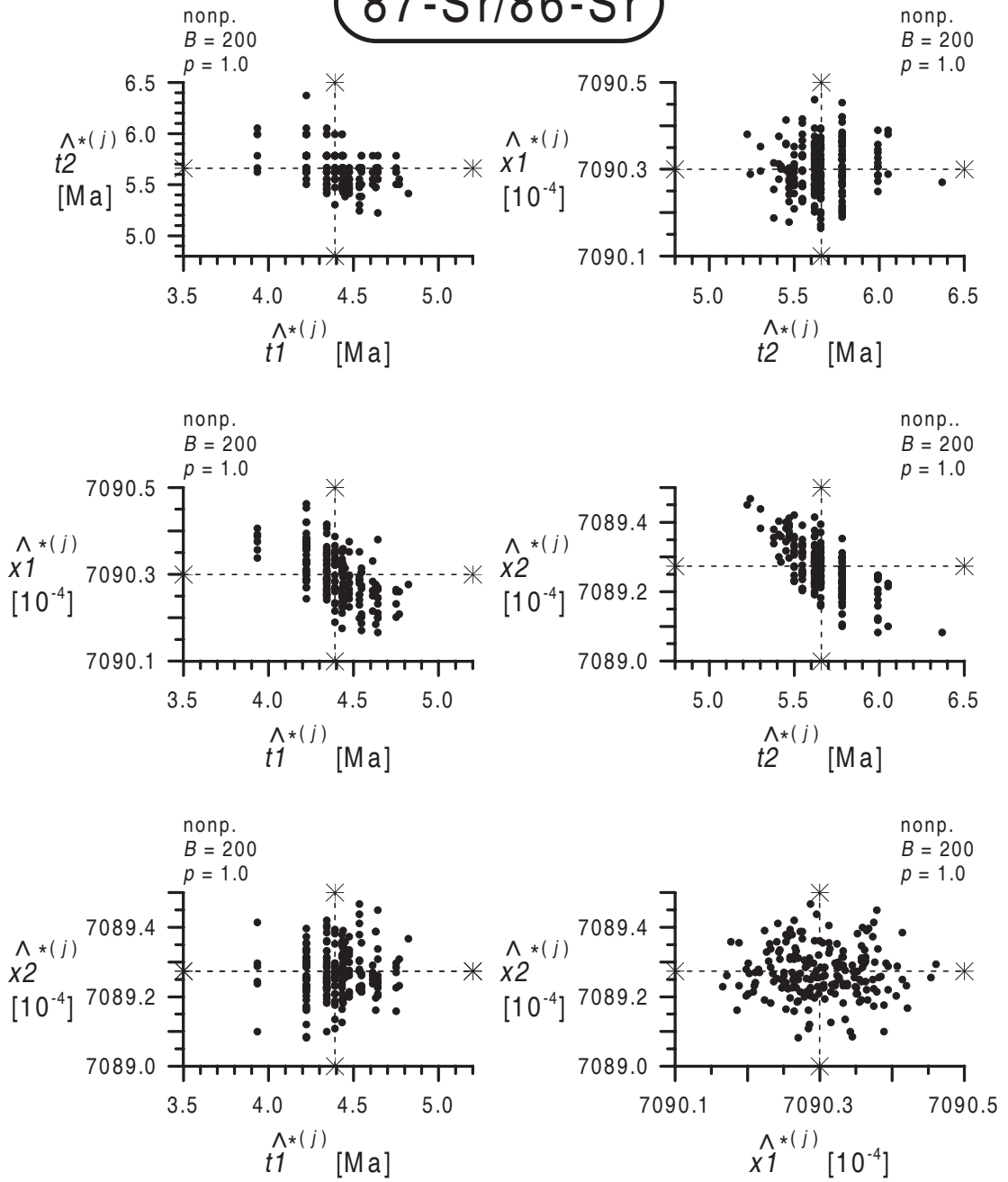
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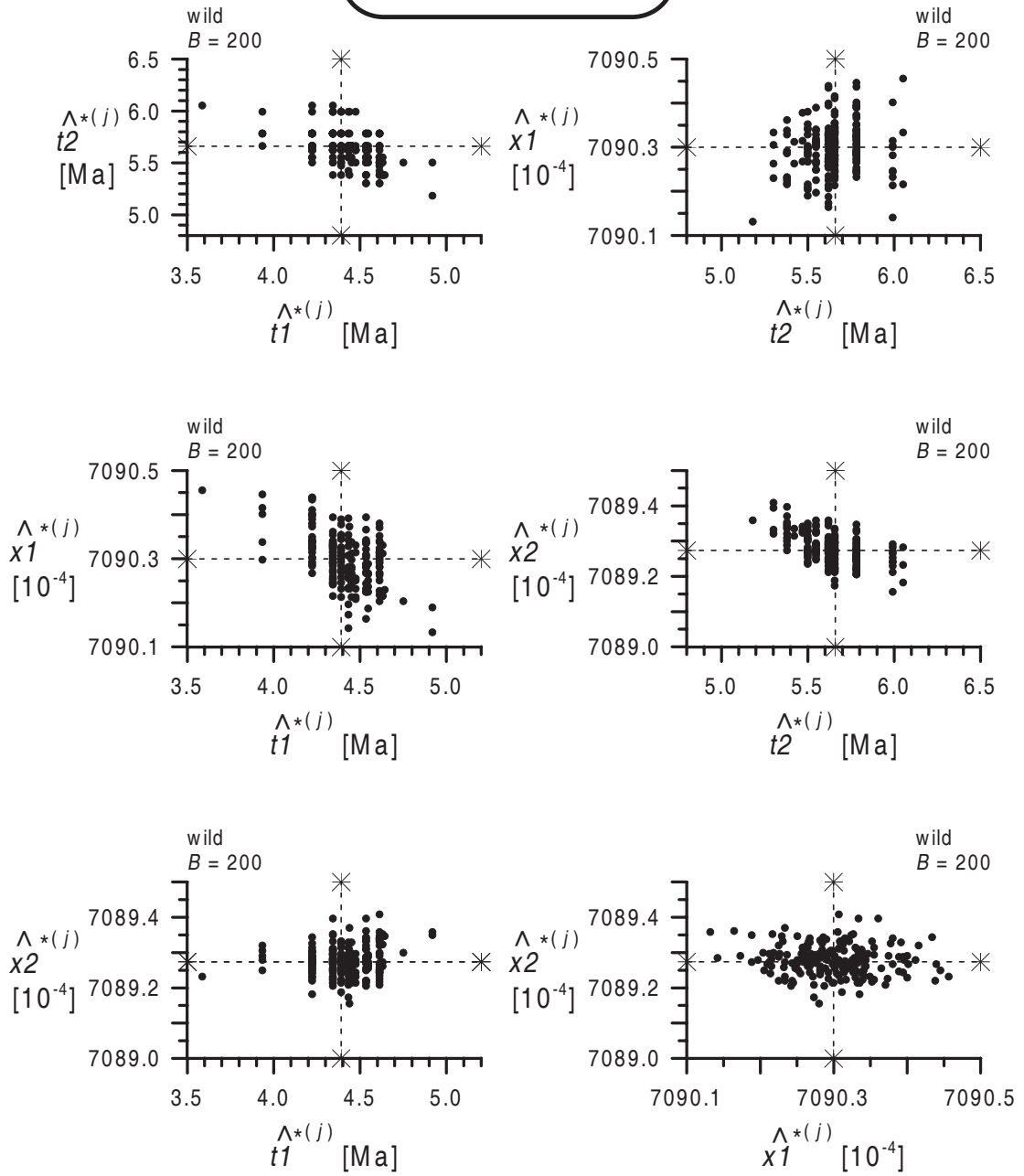
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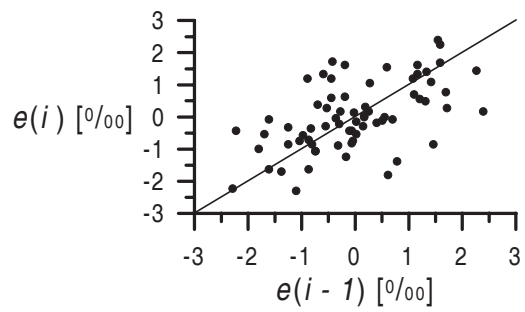
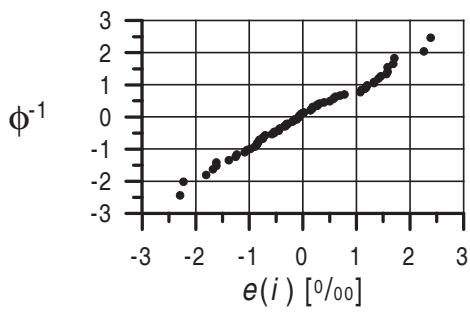
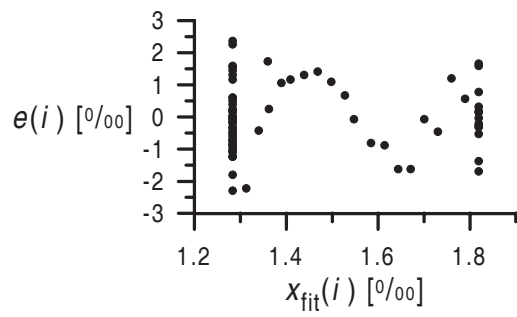
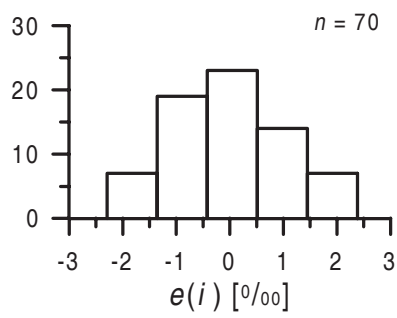
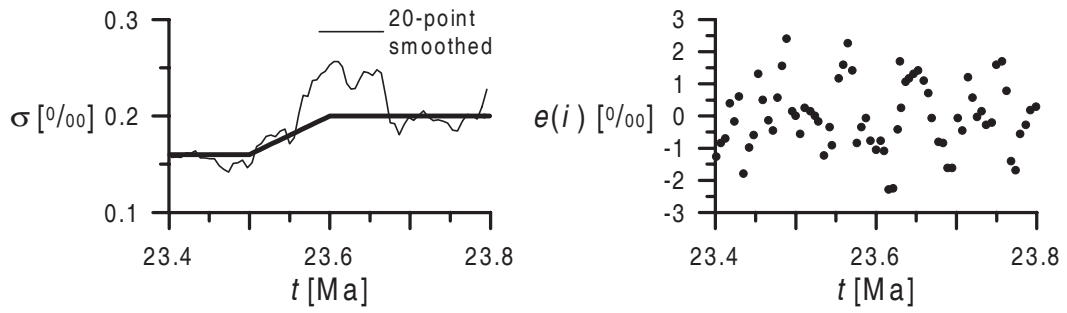
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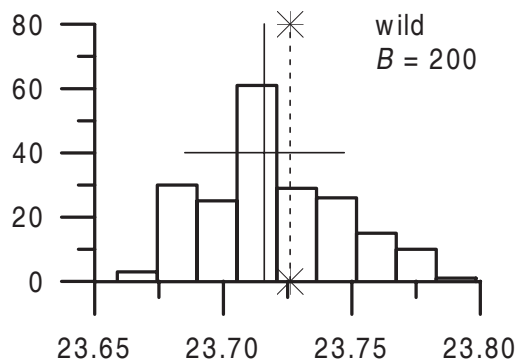
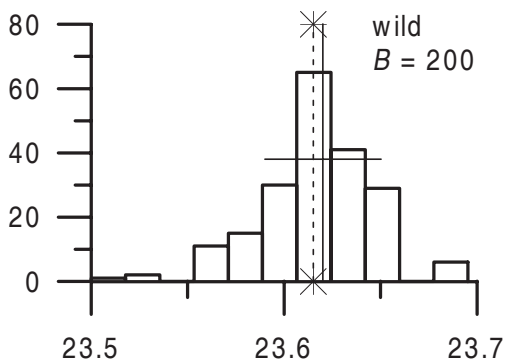
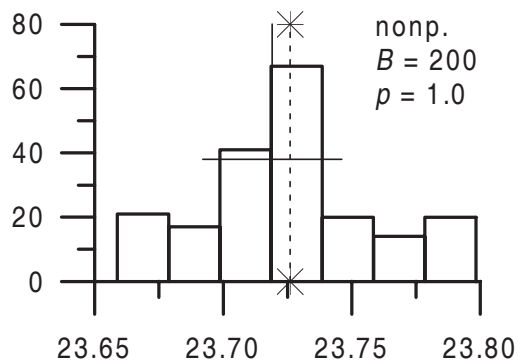
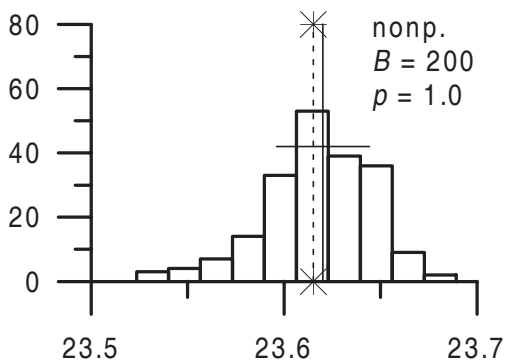
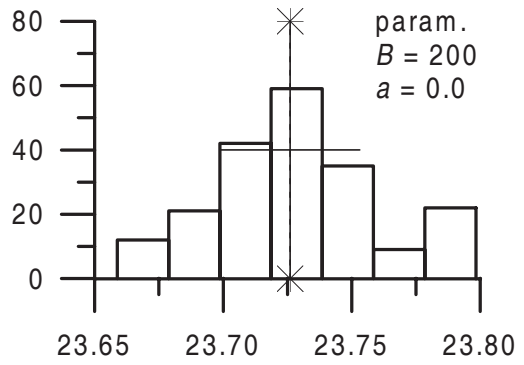
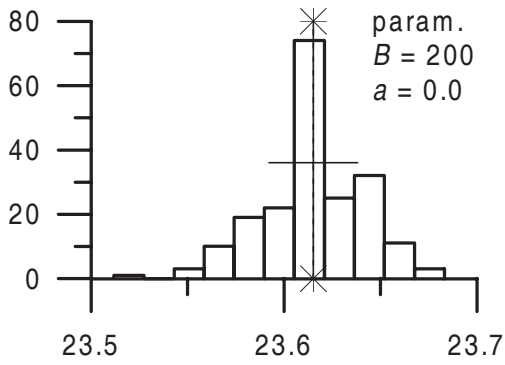
87-Sr/86-Sr



ODP 929, young part



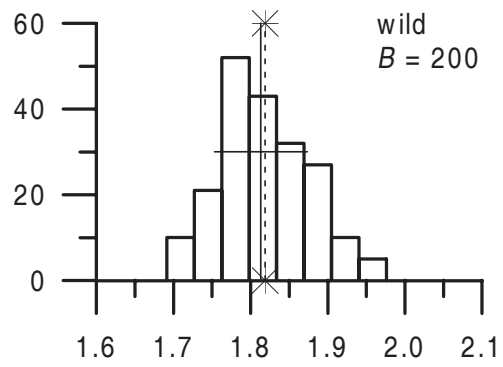
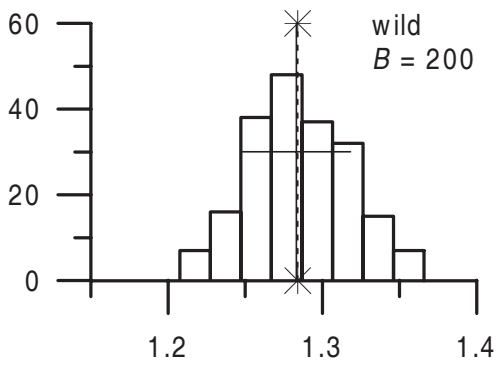
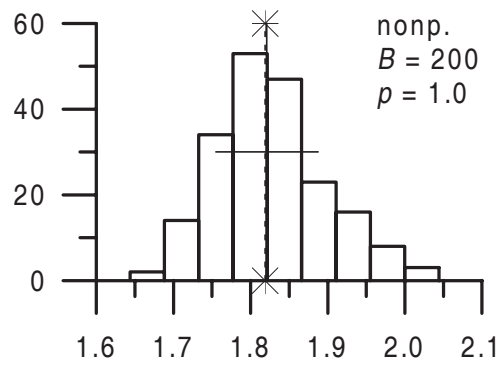
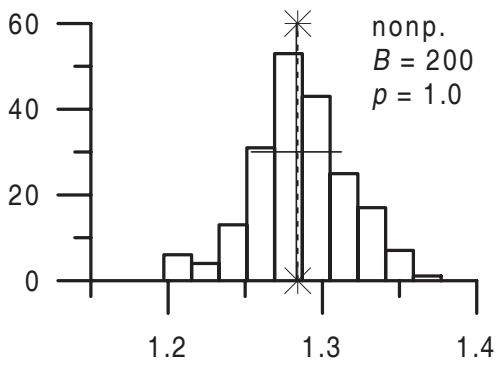
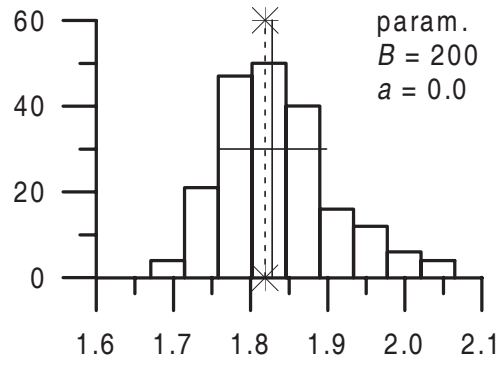
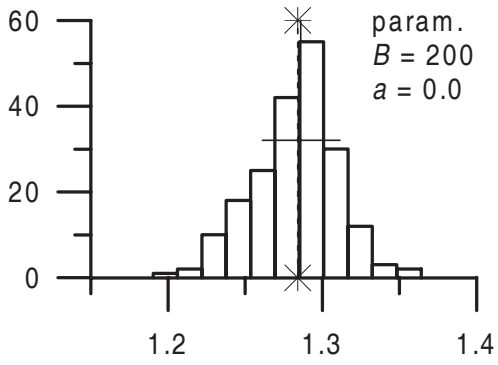
ODP 929, young part



$\Lambda^{*(j)}$
 $t1$ [Ma]

$\Lambda^{*(j)}$
 $t2$ [Ma]

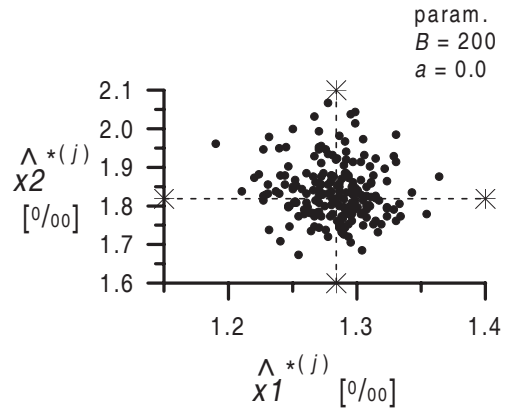
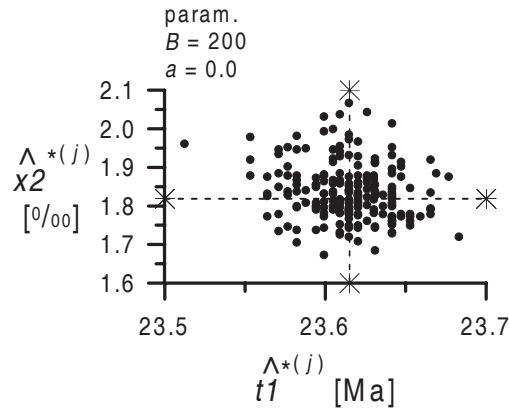
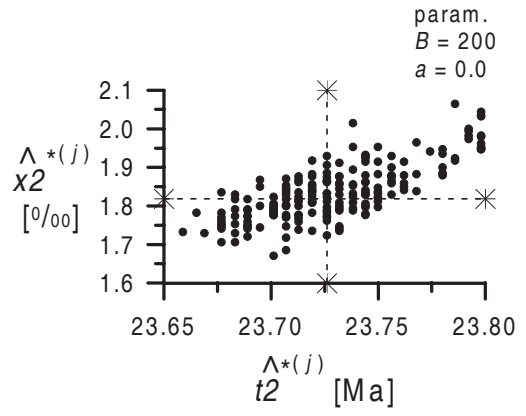
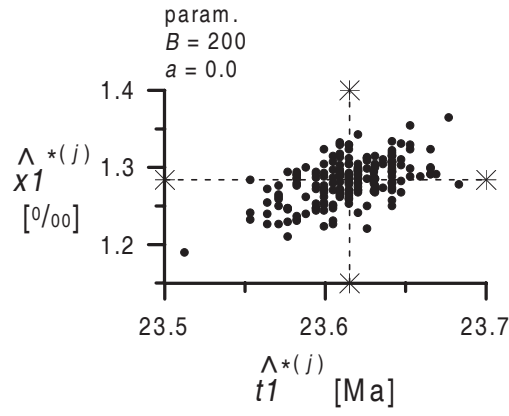
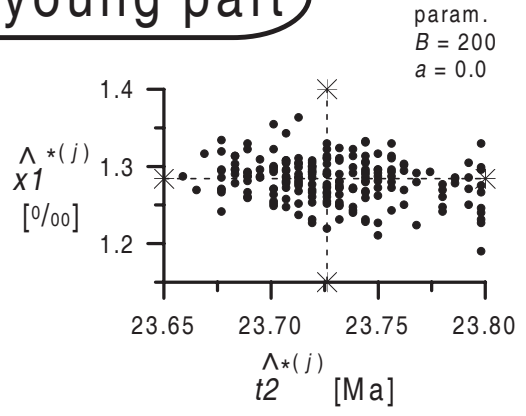
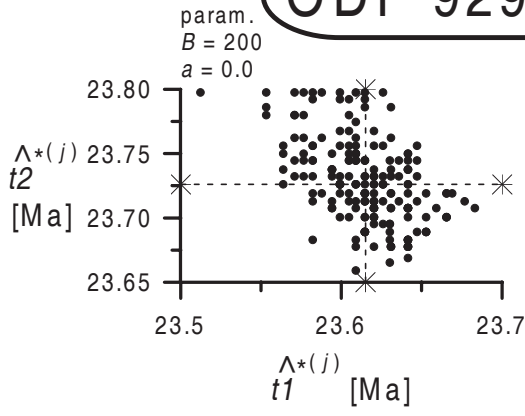
ODP 929, young part



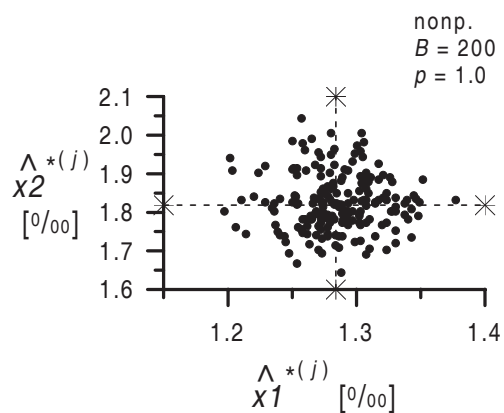
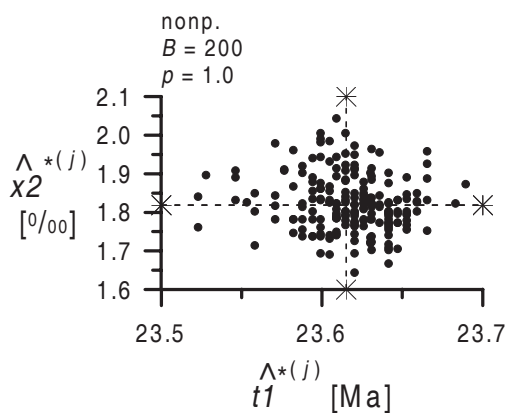
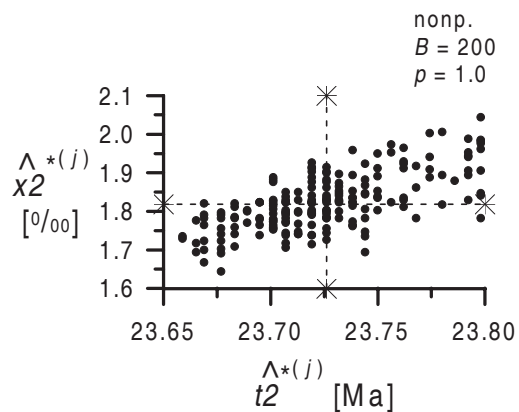
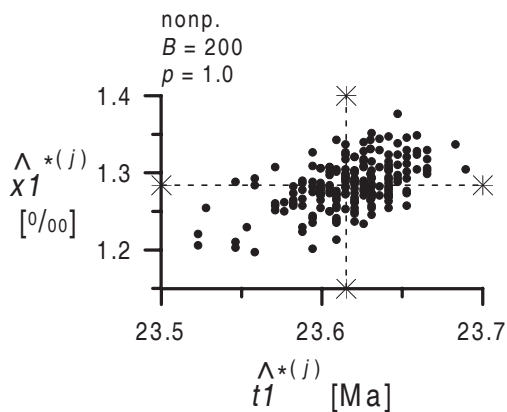
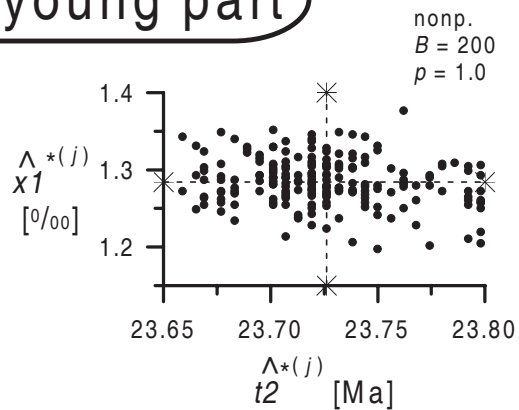
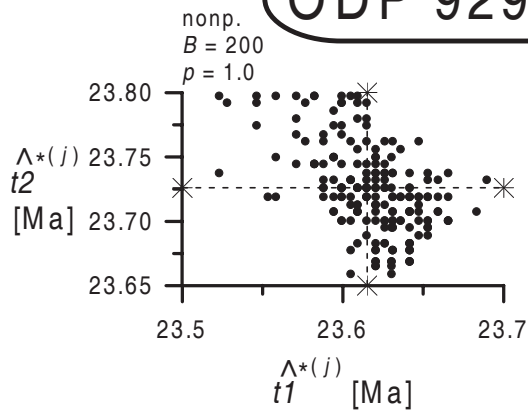
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x1 [0/00]

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x2 [0/00]

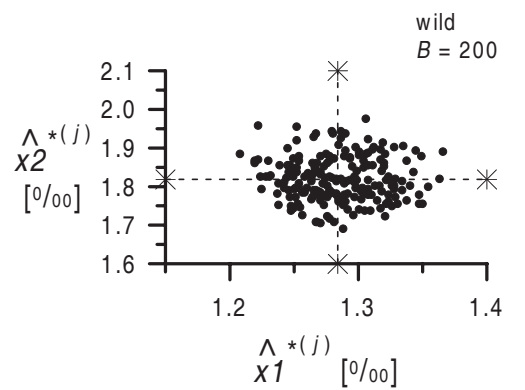
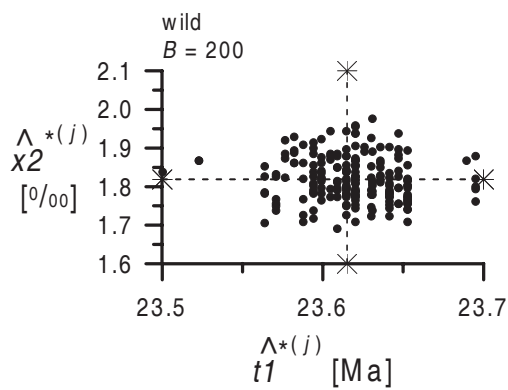
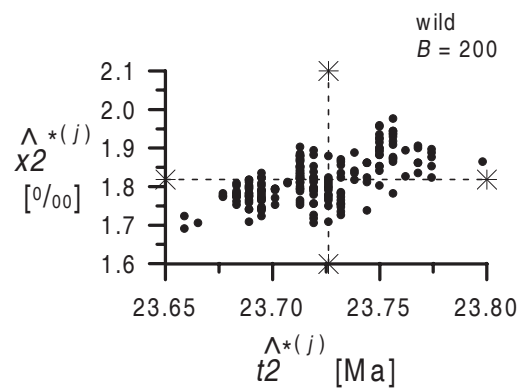
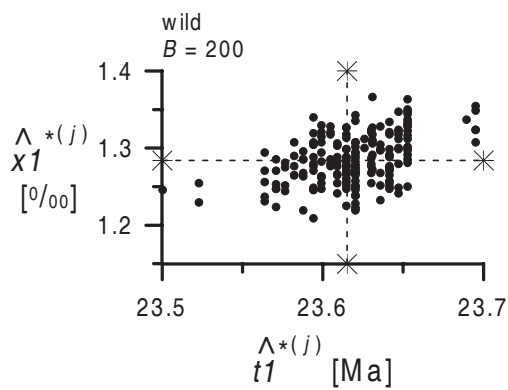
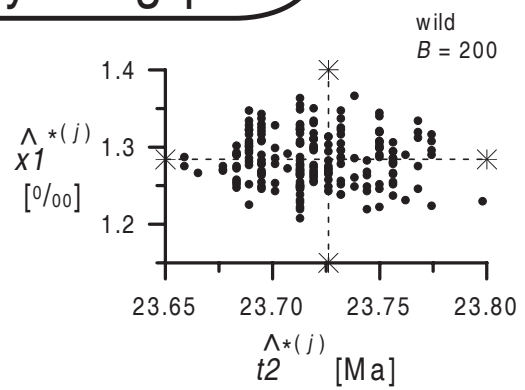
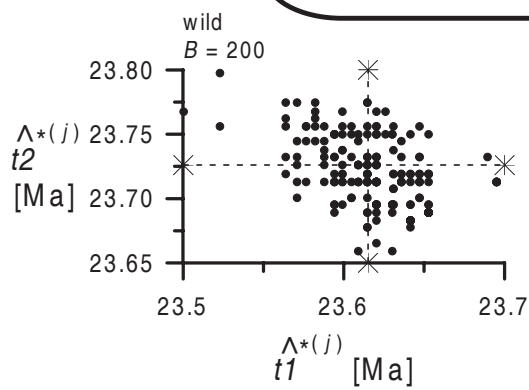
ODP 929, young part



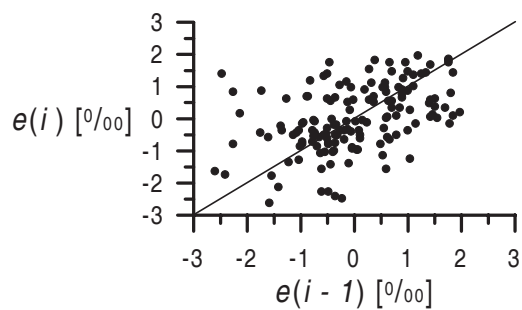
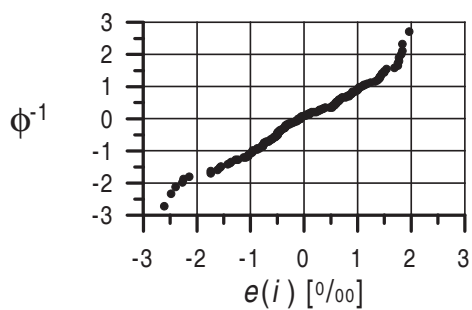
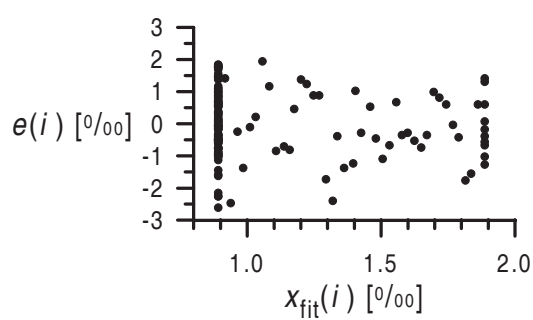
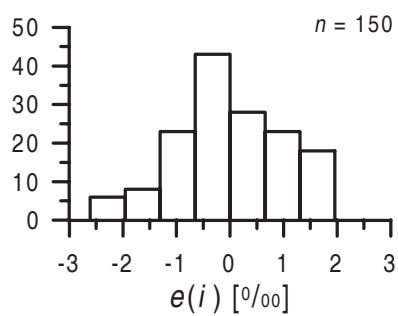
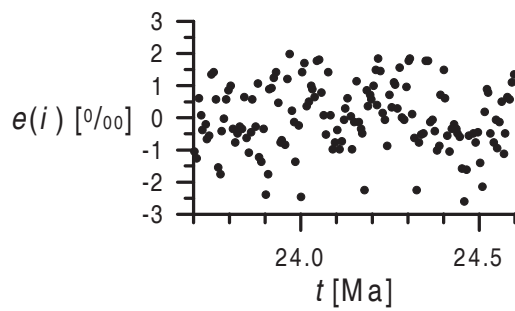
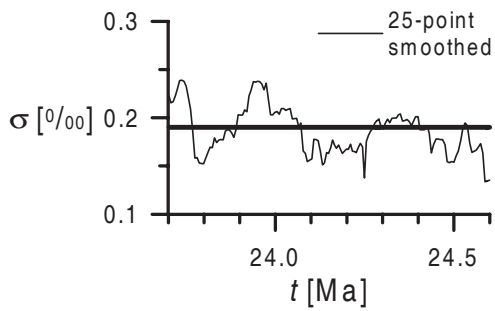
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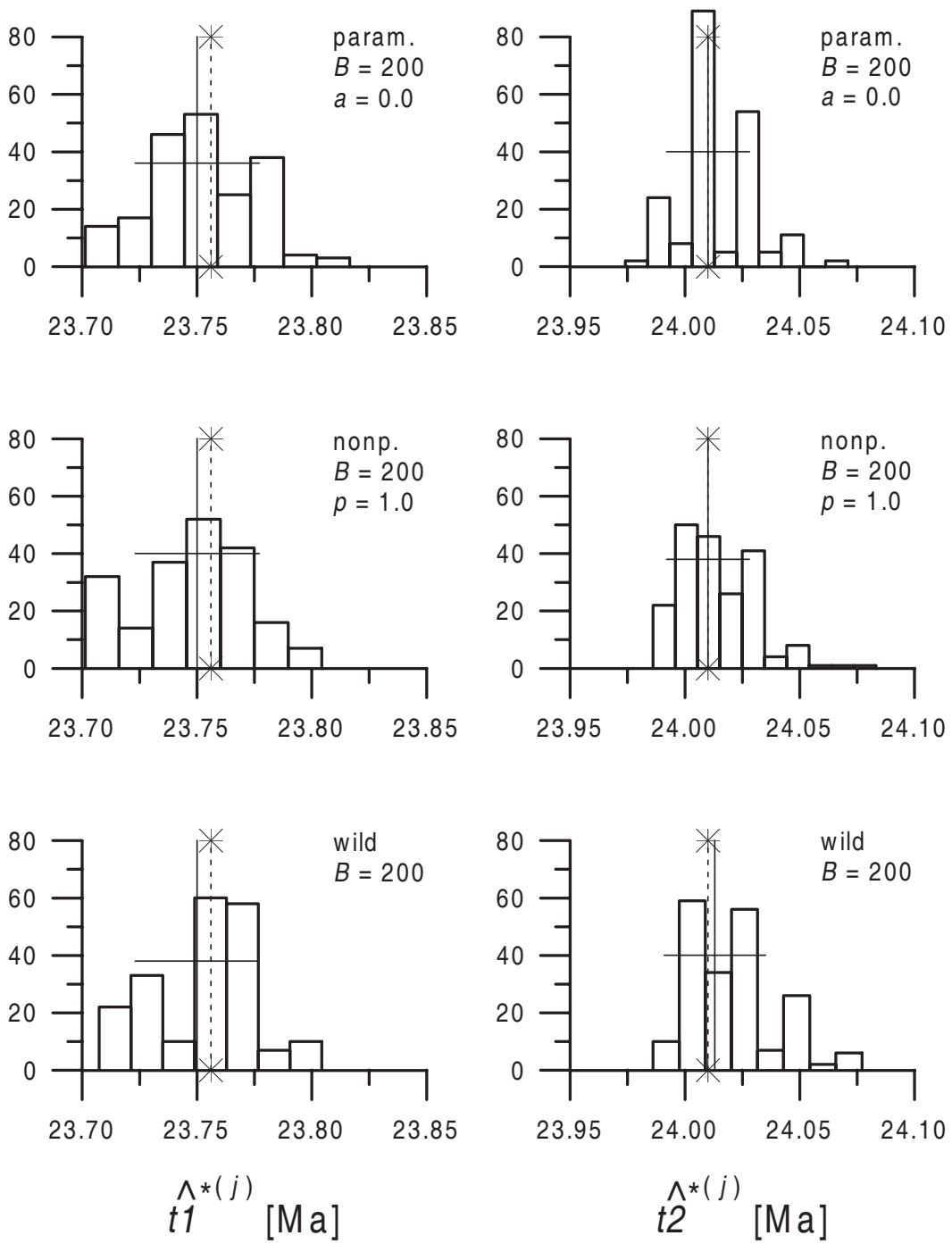
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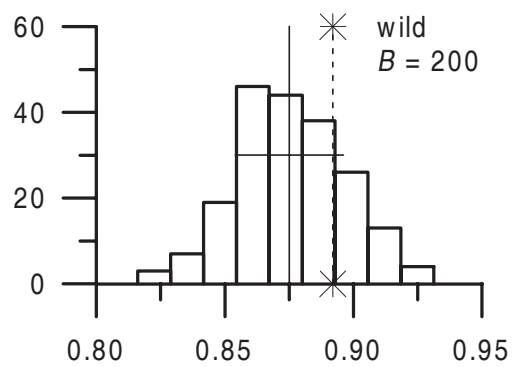
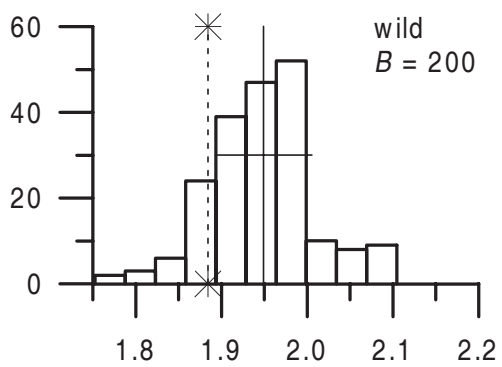
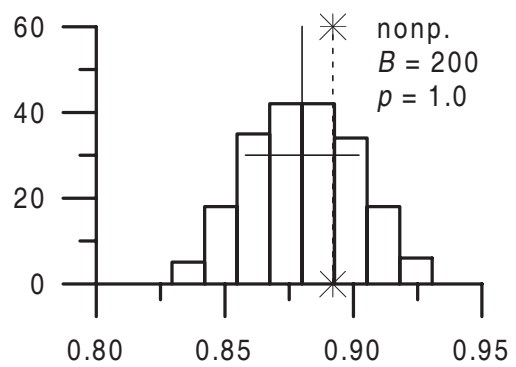
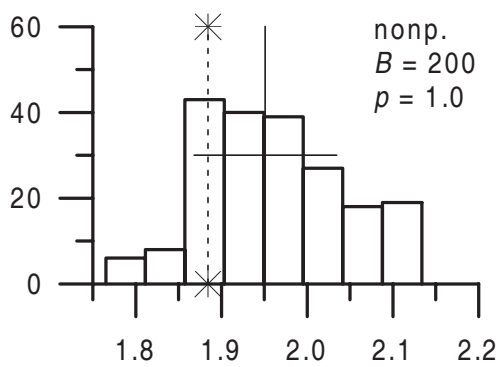
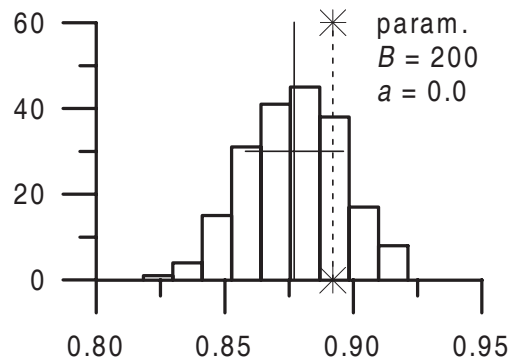
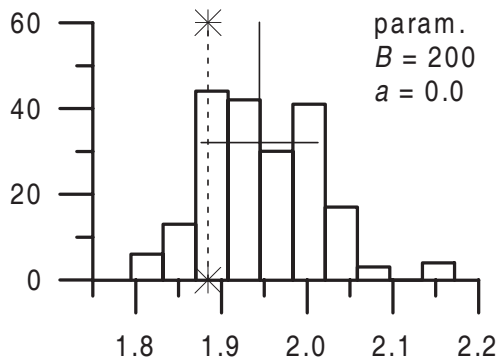
ODP 929, old part



ODP 929, old part



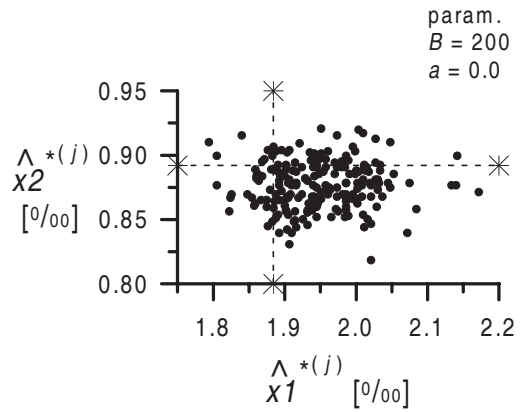
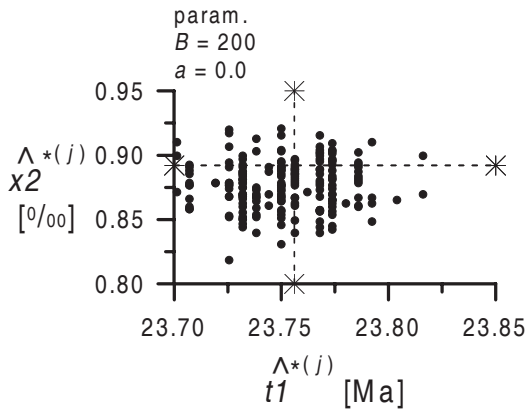
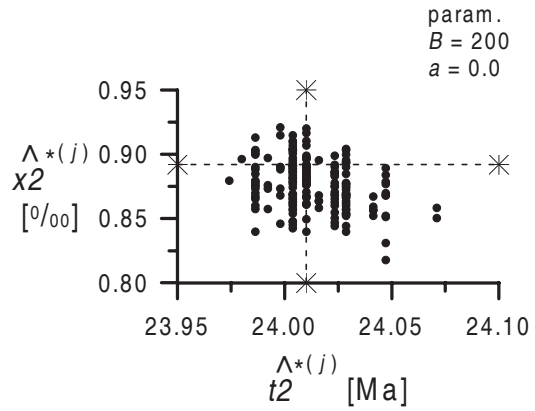
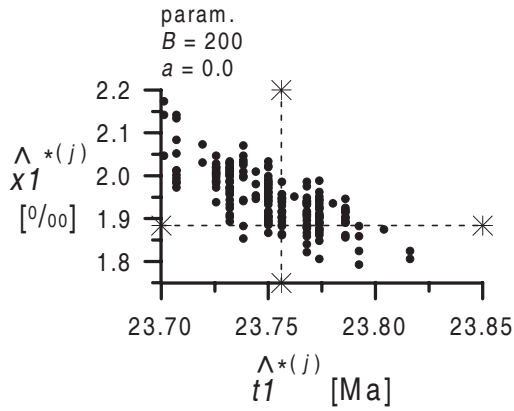
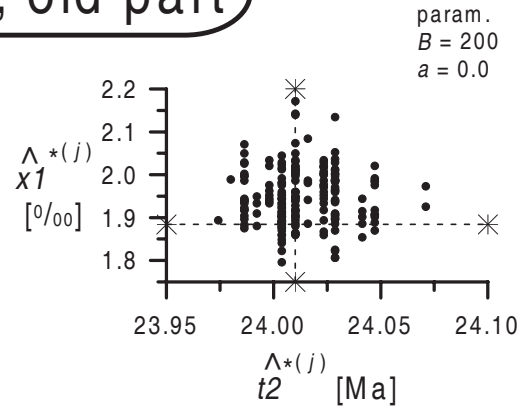
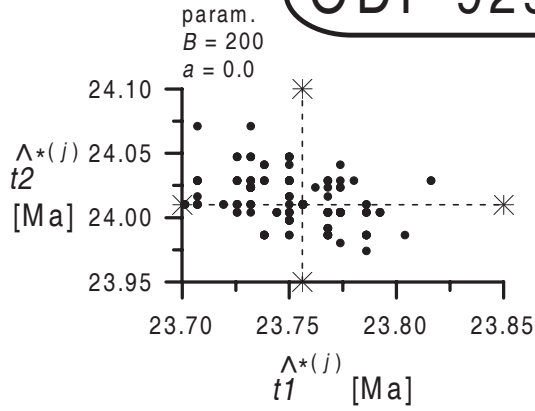
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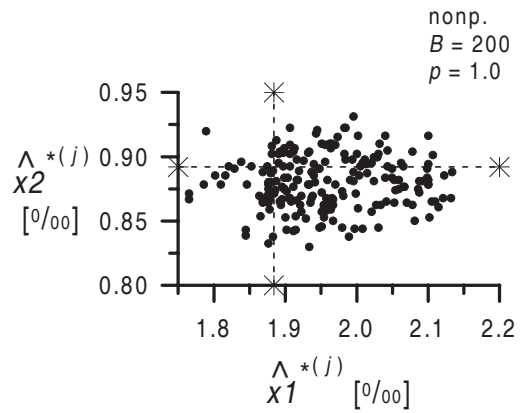
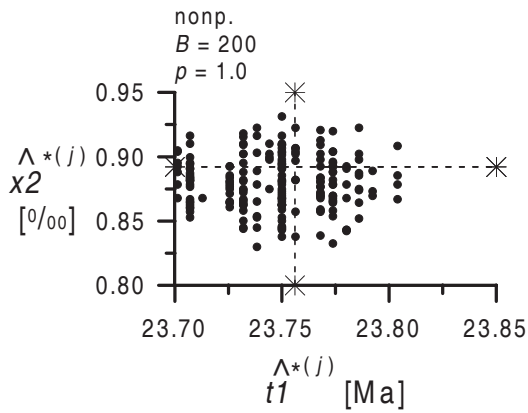
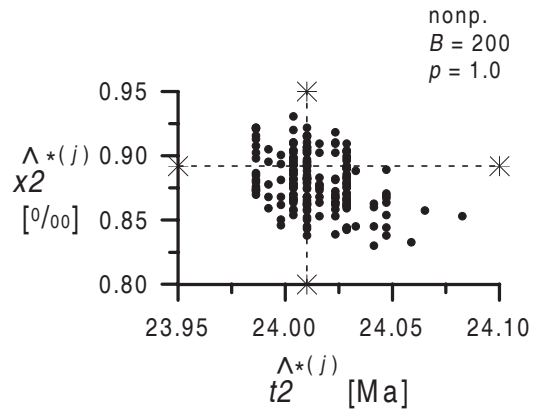
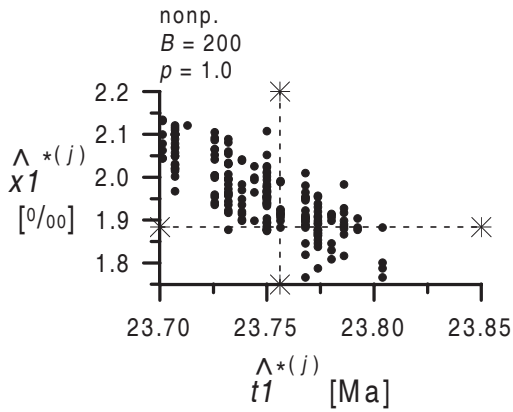
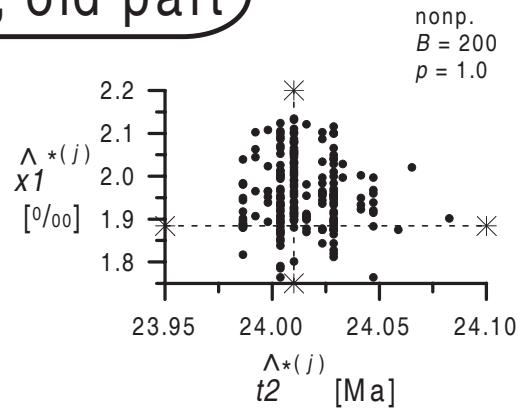
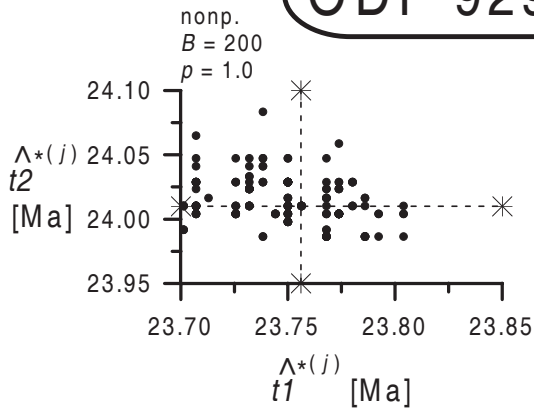
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 $x1$ [°/00]

$\Lambda^{*(j)}$
 $x2$ [°/00]

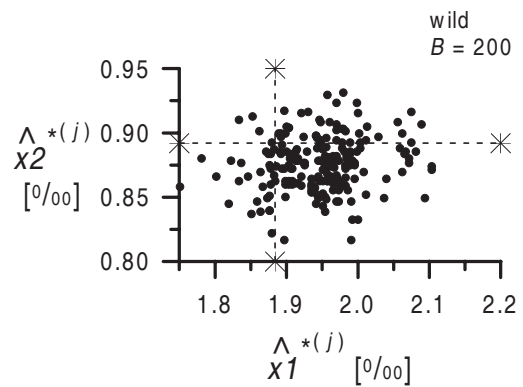
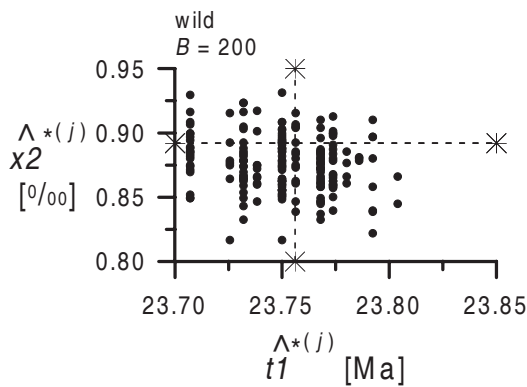
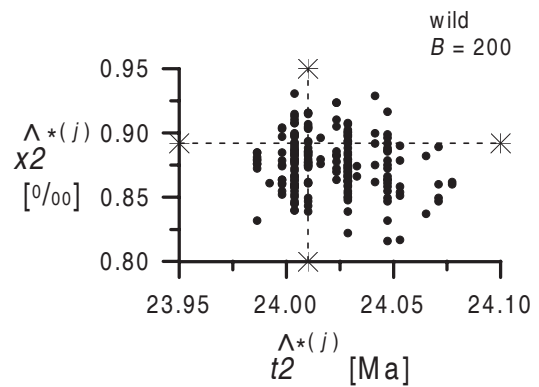
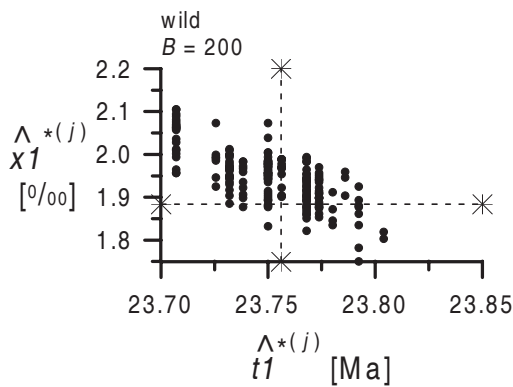
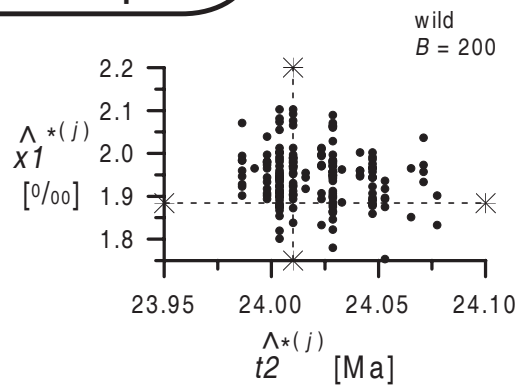
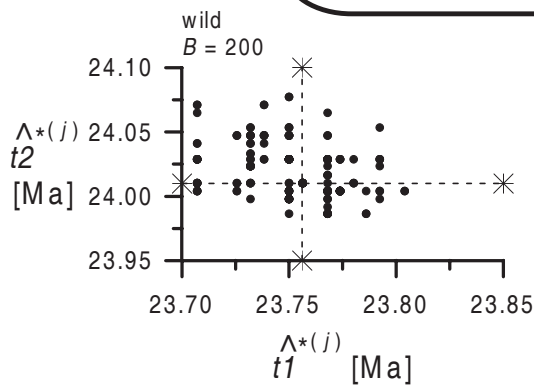
ODP 929, old part



ODP 929, old part



ODP 929, old part



B RAMPFIT Manual

RAMPFIT, written in Fortran 77, guarantees high portability. I have tested RAMPFIT on a PC with 80486-50 DX processor under MS-DOS 6.2, and on a Sun Ultra SPARC 5.5 workstation with 143 MHz clock under Unix V 4.0, but I expect it to run on other systems as well.

During execution, RAMPFIT regularly calls the gnuplot program for plotting results of current calculations on the screen; thus you need a Fortran compiler which allows external calls. (I used the Microsoft Fortran PowerStation 1.0 compiler in case of the PC, and the EPCF77 2.7 compiler in case of the workstation.)

You may request a copy of the Fortran code and hints for installation. Copies of gnuplot (version 3.6 or higher) can be obtained from the internet (for example, http://www.cs.dartmouth.edu/gnuplot_info.html).

B.1 RAMPFIT—Part 1: Detection

After you have started the program and supplied path and name of the data file (x vs. t), and the total number of points, RAMPFIT first helps to detect a transition. It plots the original time series (continue with “Enter”), and then it displays the Part 1 decision tree and the Part 1 information. That field informs about: current time interval and number of points, and (separately for x and the time spacing) the maximum, minimum, mean, and standard deviation. The decision tree allows you to choose a time interval and perform k -nearest-neighbor smoothing (in x and t together) for the mean and the standard deviation. Repeat this until you have a first guess about where a transition is located and how the time-dependent mean and standard deviation look. You proceed then to Part 2 where we take the current time interval for the regression.

B.2 RAMPFIT—Part 2: Regression

To provide the weighting for the regression, you first prescribe time-dependent standard deviation, either as a constant or as a ramp. Then you adjust the boundaries of the t_1 – t_2 search grid. RAMPFIT calculates the best fit and subsequently plots it (see Figure 3). The next plot (see

Figure 6) shows $\hat{\sigma}(t)$ (that is, the prescribed standard deviation) compared against a k -nearest-neighbor smoothing of $[x(i) - \hat{x}_{\text{fit}}(i)]$. Then the Part 2 decision tree appears with Part 2 information. That field displays: current time interval, prescribed standard deviation, $t1$ – $t2$ search boundaries, and the fit result. The latter comprises $\hat{t}1$, $\hat{x}1$, $\hat{t}2$, $\hat{x}2$, $SSQWN$, and $SSQWNS := SSQWN/[\text{average of } \hat{\sigma}(t)]$. This quantity helps to compare two fits that differ in $\hat{\sigma}(t)$. The decision tree allows new per-eye fits of $\sigma(t)$ and evaluation of new search boundaries; a new fit follows each adjustment. Repeat this until you are convinced of $\hat{\sigma}(t)$ and the $t1$ – $t2$ search grid. You proceed to Part 3 where we analyze the regression residuals. (Alternatively you may go back to Part 1 to evaluate a new time interval.)

B.3 RAMPFIT—Part 3: Residual Diagnostics

The residual plots appear in succession: histogram of $e(i)$ (see Fig. 4), normal probability plot (Fig. 5), $e(i)$ against time (Fig. 7), $e(i)$ against $x_{\text{fit}}(i)$ (Fig. 8), and $e(i)$ against $e(i - 1)$ (Fig. 9). Then the Part 3 decision tree appears, with the Part 2 information field (time interval, fit result) and the Part 3 information field. The latter displays the mean, standard deviation, and standard error of $e(i)$, and the result of the AR(1) fit, namely the values of $\hat{\tau}$ (same units as $t(i)$) and \hat{a} . The decision tree allows you to go back to Part 1 or 2. This may be necessary for achieving a properly selected time interval, an accurate fit to the standard deviation, and a well-adjusted $t1$ – $t2$ grid. Part 4, bootstrap resampling, costs the most CPU. It is recommended that the grid boundaries for the $t1$ -search ($t2$ -search) be centered around $\hat{t}1$ ($\hat{t}2$) and not be adjusted too narrow in order to keep the number of grid boundary solutions small.

B.4 RAMPFIT—Part 4: Bootstrap Resampling

The Part 4 decision tree appears with Part 3 information (residuals). This field helps to take into account serial correlation between the $e(i)$. The decision tree allows you to select among parametric, nonparametric stationary, and wild bootstrap. Each scheme demands a choice for the value of B and a seed for the random number generator. The parametric scheme additionally needs a value for the AR(1) correlation a as input; you may simply use \hat{a} . The nonparametric bootstrap requires input of a value for p . RAMPFIT suggests a value which is determined by $\hat{\tau}$ (Section 3.5). You may, however,

wish to take a smaller value which would increase the average block length.

After calculation with a particular resampling scheme, RAMPFIT gives: $\hat{t}1$, average of $\hat{t}1^{*(j)}$, standard deviation of $\hat{t}1^{*(j)}$, median, mad' , minimum, maximum, number nl of solutions on the left (lower) search boundary, and number nr of those on the right boundary; analogously for $x1$, $t2$ and $x2$. RAMPFIT also gives the results from other schemes, if such exist. Note that only the most recent results are stored for each scheme; re-running a scheme will overwrite its previous results. Now you can visualize the existing bootstrap results by selecting a scheme, a first variable ($t1$, $x1$, $t2$, $x2$), and a second variable ($t1$, $x1$, $t2$, $x2$, “no”). The value “no” leads to the bootstrap histogram for the first variable (see Figure 10), otherwise to bootstrap scatterplots (as in Fig. 11). You can repeat visualization or go back to the Part 4 decision tree. From there, you may re-run the same scheme with a new value for B , select one other scheme, go back to Part 1, 2, or 3, or exit RAMPFIT. The latter choice first allows you to change the mean smoothing and the standard deviation smoothings (for graphical reasons). Finally, RAMPFIT prints the output file names and their respective content on the screen.