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Discussion of "An analysis of global warming in the Alpine region based on nonlinear nonstationary time series models" by F. Battaglia and M. K. Protopapas

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Abstract The paper by Battaglia and Protopapas (Stat Method Appl 2012) is stimulating. It gives an elegant mathematical generalization of autoregressive models (the nine types). It explains state-of-the-art model fitting techniques (genetic algorithm combined with fitness function and least squares). It is written in a fluent and authoritative manner. Important for having a wider impact: it is accessible to non-statisticians. Finally, it has interesting results on the temperature evolution over the instrumental period (roughly the past 200 years). These merits make this paper an important contribution to applied statistics as well as climatology. As a climate researcher, coming from Physics and having had an affiliation with a statistical institute only as postdoc, I re-analyse here three data series with the aim of providing motivation for model selection and interpreting the results from the climatological perspective.

Keywords Change point model · Climate change · Instrumental period · Moving block bootstrap resampling · Temperature

1 Discussion and re-analysis

The questions Battaglia and Protopapas ask on the temperature evolution—namely about change-points in time, the rate of temperature changes and the homogeneity across measurement stations—are typical climatological questions. They remind us of the first part of the definition (Brückner 1890; Hann 1901; Köppen 1923) of

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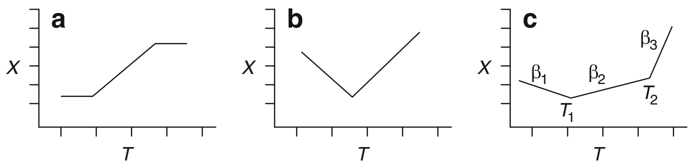

Fig. 1 Piecewise-linear regression models, **a** ramp, **b** break and **c** two-break/three-regime. T is time and X is a (climate) variable. The latter model, used here, has its slope ($\beta_1, \beta_2, \beta_3$) and change-point time (T_1, T_2) parameters denoted

climate, namely in terms of mean or trend. We should recognize the second part of the definition, in terms of variance, which may stimulate future statistical work on heteroscedastic models. The debate about climate-risk changes, amplified by the contribution of Working Group I to the Fourth Assessment Report of the IPCC (Solomon et al. 2007), should let us think about adding a third part to the definition (in terms of extremes).

My own approach (Mudelsee 2000, 2009, 2010) to dealing with the research questions about change-points in a climate variable is formulating piecewise-linear regression models (Fig. 1). Fitting such models to time series data $\{t(i), x(i)\}_{i=1}^{n}$ can be achieved at reasonable computing costs by combining a least-squares criterion with a brute-force search for the change-point times.

I extend the break model to the two-break/three-regime model (Fig. 1) to compare results with those from linear-level, PLTAR-time fitting in the paper by Battaglia and Protopapas (2012), Table 4 and Fig. 6 therein. This is done for the temperature time series from Bern, Karlsruhe and Strasbourg. The model is

$$X(i) = X_{\text{two-break}}(i) + S \cdot X_{\text{noise}}(i), \tag{1}$$

where $X(i)$ is the climate variable in discrete time, $X_{\text{two-break}}(i)$ is the trend component (Fig. 1c), S is the variability, assumed constant (homoscedastic climate), and $X_{\text{noise}}(i)$ is the zero-mean, unit-standard deviation noise component. Least-squares estimation minimizes

$$\sum_{i=1}^{n} [x(i) - x_{\text{two-break}}(i)]^2 , \tag{2}$$

where $x_{\text{two-break}}(i)$ is the sample version of $X_{\text{two-break}}(i)$. Since I have not ready a software implementation of a brute-force search for both change-points $\widehat{T}_1$ and $\widehat{T}_2$ (Fig. 1c), but only for a single change-point (Mudelsee 2009), I use the result from the paper by Battaglia and Protopapas (2012), Table 4 therein, and minimize two sums for the break model (Fig. 1b),

$$\sum_{i=1}^{n_1} [x(i) - x_{\text{break}}(i)]^2 , \quad \sum_{i=n_2}^{n} [x(i) - x_{\text{break}}(i)]^2 , \tag{3}$$

Table 1 Results of three-regime break function regression for temperature series from Bern, Karlsruhe and Strasbourg

Station	$\widehat{\beta}_1$	$\widehat{T}_1$	$\widehat{\beta}_2$	$\widehat{T}_2$	$\widehat{\beta}_3$	R^2
Bern	-0.01 ± 0.04	1891 ± 21	0.12 ± 0.03	1984 ± 9	0.61 ± 0.30	0.44
Karlsruhe	-0.03 ± 0.03	1890 ± 16	0.11 ± 0.02	1984 ± 7	0.68 ± 0.29	0.41
Strasbourg	-0.03 ± 0.04	1889 ± 19	0.11 ± 0.02	1986 ± 7	0.75 ± 0.35	0.37

Given are slopes (in degrees/decade) within a regime, the change-point times between two regimes and the coefficient of determination. Error bars are normalized MAD values (corresponding to 1-σ standard errors in case of normal distributions) obtained from $B = 2000$ moving-block bootstrap resamplings

where $t(n_1)$ is the estimated (Battaglia and Protopapas 2012) later change-point time and $t(n_2)$ is the estimated (Battaglia and Protopapas 2012) earlier change-point time. My results (Table 1) agree almost perfectly to what is given in the paper (Battaglia and Protopapas 2012), Tables 4 and 5 therein (but note the deviation in slope of the third regime for Strasbourg). This agreement basically supports the genetic estimation routine (Battaglia and Protopapas 2012), which gives same results as the brute-force (optimal) search adopted by me.

One drawback of the PLTAR/STAR model approach as presented (Battaglia and Protopapas 2012), is that no uncertainty measures are provided. The statement that the significance of the slope estimates "may be evaluated approximately by means of standard least squares theory" is insufficient to satisfy the practitioner. Climate data, as any measured data, are limited in size and influenced by measurement error. Approximate ($n \rightarrow \infty$) results may not hold for the sizes ($n \approx 200$) of the temperature series. Furthermore, standard least squares theory, in fact likely: any theory, is incapable of allowing to derive uncertainty measures of the estimated change-point times.

On the other hand, uncertainty measures can be constructed for the break-model estimation (1–2) by means of bootstrap resampling (Efron and Tibshirani 1993; Mudelsee 2010). The regression residuals $e(i)$ are given by

$$e(i) = x(i) - \widehat{x}_{\text{two-break}}(i), \tag{4}$$

where $\widehat{x}_{\text{two-break}}(i)$ is the two-break/three-regime model with estimated parameters on the sample level. Moving overlapping blocks are resampled from the residuals to take autocorrelation into account and to be independent of distributional assumptions (Künsch 1989). [Block-length selection is based on the autocorrelation coefficient and the sample size (Carlstein 1986; Sherman et al. 1998; Mudelsee 2010).] These bootstrapped residuals, added to the fit curve, form the resample, on which fitting is repeated, yielding replications of fit parameters (e.g., $\widehat{T}_1$). The procedure resampling-fitting is carried out $B = 2000$ times. The resulting bootstrap histograms for $\widehat{T}_1$ and $\widehat{T}_2$ display the location and spread of the replications (Fig. 2). Since the distributions exhibit skewness, I report not the standard error but its robust counterpart, MAD/0.6745, where MAD is the median of absolute distances to the median (Tukey 1977). The results (Table 1) show "robust standard errors" of somewhat less than

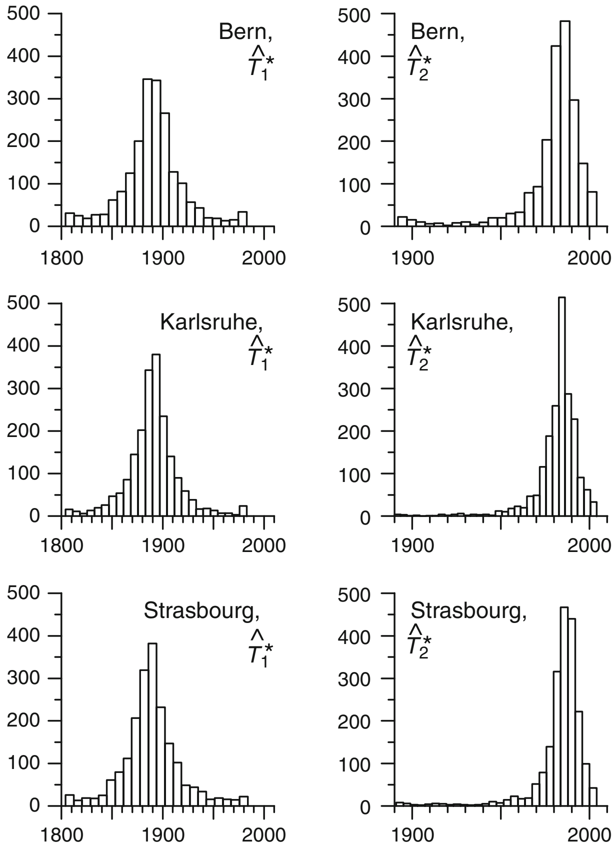

Fig. 2 Bootstrap histograms of change-point time replications (denoted by *asterisk*)

10 years for the later change-point, $\widehat{T}_2$, and around 20 years for the earlier change-point, $\widehat{T}_1$.

1.1 Model selection

Not only statisticians, also we climatologists follow usually the rational approach (Popper 1935) and describe a variable as consisting of deterministic and random components. Also we try to capture on basis of the data as much as possible of the "climate essence" using a parsimonious description. Luckily, there is enough room to formulate different model types. This allows for partly subjective views of the same data through different paradigms (Kuhn 1970), enabling scientific progress. Climatologists' view is also influenced by prior physical knowledge, which helps further in selecting models. For example, physics argues against selecting an ARIMA model without bounds because climate "does not run away," and it is reassuring that the present paper (Battaglia and Protopapas 2012) does not support the ARIMA view. Room allows also to "play" with the distribution of what is put into trend and what into noise. My description (1) adopts more complex (two-break/three-regime) trend and more simple noise components. [Theoretical climatology motivates the simple first-order autoregressive noise model (Hasselmann 1976).] The description by Battaglia and Protopapas (2012) ignored writing a deterministic trend component and put complexity into the noise. An advantage of their description (Battaglia and Protopapas 2012) is that, if other series are to be analysed, more complex "trend" forms (more change-points in time, also change-points in level) are automatically included. An advantage of my description is a foundation on physics and the applicability also to series unevenly spaced in time (embedding problem avoided). (Uneven spacing is found in paleoclimatology, where natural climate archives are sampled).

1.2 Interpretation of results: accelerated warming

The original results obtained by Battaglia and Protopapas (2012), Tables 4 to 6 and Figures 6 to 9 therein, supported and assigned with error bars for three stations in this discussion (Table 1), show unequivocally that regional warming in the greater Alpine area accelerated considerably during the 1980s. The concept of accelerated warming has already been in the mind of climatologists, as Fig. TS.6 from the IPCC report (Solomon et al. 2007) demonstrates. That figure displays annual global mean temperatures over 1850–2005 and linear regression lines for a selection of time spans (all ending 2005). The slopes of the regressions get steeper when more recent spans are analysed. The paper under discussion (Battaglia and Protopapas 2012) employs a more suitable tool than the IPCC has used, and it delivers firm statistical evidence (change-point values, slopes) for the accelerated warming. The next step would be to perform such change-point analyses for globally distributed stations. The emerging pattern would shed light on external climate forcing or internal climate feedback mechanisms, which may act on regional spatial scales, and enhance our understanding of the ongoing climatic change.

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